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INTERACTIVE FUZZY META-GOAL PROGRAMMING FOR PRODUCTION PLANNING IN INDUSTRY

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ABSTRACT

This paper introduced an interactive fuzzy meta-goal programming for production planning in industry. The decision-maker first propose goals toward his/her benefits for the problem, and then sets the target values for the meta-goals, like aggregate achievement, maximum deviation and number of unsatisfied goals assuming these target values being imprecise in nature. Flexibility is one of the most important features of the interactive meta-goal programming. This approach is proposed with the help of an example taken from the production plant. Further, LINDO 15.0 optimizer solver is used to draw results of the problem.

Production planning is a complicated task. It requires cooperation among different functional units in any organization. A good understanding in terms of customers, products and manufacturing processes is a must in order to have an efficient production planning [10]. Planning involves a hierarchy of decisions dealing with different issues in the manufacturing environment [11]. Therefore, new tools for production planning are required that consider these issues. To overcome this challenge, this paper proposes a decision-support framework based on Interactive Fuzzy Meta-Goal Programming (IFMGP) approach. The meta-goal programming framework proposed by Rodriguez et al. [12] allows the decision maker more flexibility in expressing their preferences by means of setting meta-goals. These can be thought of as secondary goals derived from the original set of goals. IMGP helps in formulating the Meta-GP approach within an interactive framework. Thus, ‘‘satisficing’’ targets are allocated to each attribute and the meta-achievement function is selected in accordance with the DM’s actual preferences. Here, the decision maker can establish target values on several achievement functions. Further, such target values can be assumed fuzzy in nature and then apply an interactive procedure to update these values. The use of fuzzy set theory in GP was first considered by Narasimhan [7, 8, 9], Hannan [2, 3, 4], Ignizio [5]. Rubin and Narsimhan [13] and Tiwari et al. [14, 15] have investigated various aspects of decision problem using FGP. Yaghoobi at el. [16] apply the conventional MINMAX approach in goal programming to solve fuzzy goal programming. An extensive review of these papers is given by Tiwari et al. in 1985.A hypothetical example is given to justify the proposed framework.

I. IFMGP-BASED DECISION ANALYSIS FRAMEWORK

Before discussion of the IMGP-based decision analysis framework let us begin with the fundamental concepts of Goal Programming (GP). It was proposed in 1961 by Charnes and Cooper [1] and was introduced as an extension to Linear Programming. In one of its most general forms, GP model is formulated as:

$$\text{Minimize: } Z = \sum_{i=1}^{r} \left(w_i^n n_i + w_i^p p_i \right) \tag{1}$$

Subject to:

Goal functions

$$\sum_{j=1}^{s} a_{i,j} x_j + n_i - p_i = T_i \text{ , for } i = 1, 2, ..., r \tag{2}$$

Hard constraints

$$\sum_{j=1}^{s} b_{k,j} x_j \le C_k \text{ , for } k = 1, 2, ..., t \tag{3}$$

Non-negative constraints

$$x_j, n_i, p_i, w_i^n, w_i^p \ge 0 \quad \text{for } i = 1, 2, ..., r; \text{ for } j = 1, 2, ..., s; \text{ for k} = 1,2,...,\text{t} .$$

A per unit penalty of either u_i or v_i for every unit of unwanted deviation from the target value is assumed in standard GP. This lead to a linear relationship between the magnitude of the unwanted deviation and the penalty imposed. Introducing the penalty function and normalization constant in the above goal programming model, the modified GP model is defined as :

$$\text{Minimize: } Z = \sum_{i=1}^{r} \left(\frac{\sum_{u=1}^{h} (w_i^n n_{iu} + w_i^p p_{iu})}{T_{i1}} \right) \tag{4}$$

Subject to:

Goal functions

$$\sum_{j=1}^{u} a_{i,j} x_j + n_{iu} - p_{iu} = T_{iu} \text{ , for } i = 1, 2, ..., r; \text{u} = 1, 2..., \text{h} \tag{5}$$

Hard constraints

$$\sum_{j=1}^{s} b_{k,j} x_j \le C_k \text{ , for } k = 1, 2, ..., t \tag{6}$$

$$\sum_{j=1}^{s} a_{i,j} x_j \geq A \tag{7}$$

Non-negative constraints

$$x_j, n_i, p_i, w_i^n, w_i^p \geq 0 \tag{8}$$

for $i = 1, 2, ..., r;$ for $j = 1, 2, ..., s;$ for k = 1,2,...,t; for $s = 1, 2..., k$

To mathematically model IFMGP based decision problems, the following notations are introduced to represent the parameters and variables involved in the model.

r	Total number of goal functions
s	Total number of decision variables
t	Total number of hard constraints
i	Goal function index, $i = 1, 2, ..., r$
j	Decision variable index, $j = 1, 2, ..., s$
k	Hard constraint index, $k = 1, 2, ..., t$
h	Total penalty function
w_i^n, w_i^p	Relative weighting factors assigned to the slack and surplus goal deviations of i^{th} goal function respectively
n_{iu}, p_{iu}	Slack and surplus goal deviations of i^{th} goal function respectively with penalty u
a_{ij}	Technology coefficient for the j^{th} decision variable of i
u	Penalty attached to i^{th} goal function;
A	Minimum target level attached to the function below which the function is invalid
x_j	j^{th} decision variable
T_{iu}	Target value of i^{th} goal function with penalty
$b_{k,j}$	Technology coefficient for the j^{th} decision variable of the k^{th} hard constraint
C_k	Constraints value of the k^{th} hard constraint
l	l^{th} highest priority level, index for the current sub-problem of the overall meta-goal programming decision problem
ρ	Priority level index for all levels of higher priority than the l^{th} level, $\rho = 1, ..., l-1$
r^l, r^p	Total number of goal functions in the l^{th} and ρ^{th} priority level respectively
$a_{i,j}^l, a_{i,j}^\rho$	Technology coefficient for the j^{th} decision variable of the i^{th} goal function in the l^{th} and ρ^{th} priority level respectively

$T_{iu}^{l}, T_{iu}^{\rho}$	Target value of i^{th} goal function in the l^{th} and ρ^{th} priority level respectively with penalty u
$w_{iu}^{l}, w_{iu}^{\rho}$	Relative weighting factors assigned to the undesired goal deviations of i^{th} goal function in the l^{th} and ρ^{th} priority level respectively with penalty u
$n_{iu}^{l}, n_{iu}^{\rho}$	Slack goal deviations of i^{th} goal function in the l^{th} and ρ^{th} priority level respectively with penalty u
$p_{iu}^{l}, p_{iu}^{\rho}$	Surplus goal deviations of i^{th} goal function in the l^{th} and ρ^{th} priority level respectively with penalty u
$d_{iu}^{l}, \mathrm{d}_{iu}^{\rho}$	Undesired goal deviations of i^{th} goal function in the l^{th} and ρ^{th} priority level respectively with penalty u
$\alpha_{1,2,3}^{l}, \alpha_{1,2,3}^{\rho}$	Slack deviation of meta-goal variant-1, 2, or 3 in the l^{th} and ρ^{th} priority level respectively
$\beta_{1,2,3}^{l}, \beta_{1,2,3}^{\rho}$	Surplus deviation of meta-goal variant-1, 2, or 3 in the l^{th} and ρ^{th} priority level respectively
$Q_{1,2,3}^{l}, Q_{1,2,3}^{\rho}$	Target value of meta-goal variant-1, 2, or 3 in the l^{th} and ρ^{th} priority level respectively
D^{l}, D^{ρ}	Constraining variable for meta-goal variant-2 in the l^{th} and ρ^{th} priority level respectively, they represent the maximum allowed goal deviations
M_{i}^{l}, M_{i}^{ρ}	Large enough positive constraining variable for meta-goal variant-3 in the l^{th} and ρ^{th} priority level respectively
y_{i}^{l}, y_{i}^{ρ}	Binary variable for meta-goal variant-3 in the l^{th} and ρ^{th} priority level respectively
$\mu_{1,2,3}$	Fuzzy meta goals membership functions
$R_{1,2,3}^{\rho}$	Achievement of the fuzzy meta-goal variant-1, 2, or 3 membership function value accepted by decision makers in a higher level ρ

Equations (9) – (33) represent the meta-goal programming decision problem for priority level l. To solve the entire decision problem model, priority levels are analyzed one at a time, from the highest prioritized to the lowest. Meta-goal solutions obtained from levels higher than are converted into hard constraints during the analysis of level l.

This ensures higher prioritized goals can be best achieved before the lower ones.

$$\text{Maximize: } Z = \mu_1 + \mu_2 + \mu_3 \tag{9}$$

Subject to:

Hard constraints

$$\sum_{j=1}^{s} b_{k,j} x_j \leq C_k, \text{for } k = 1,2,...,t \tag{10}$$

$$\sum_{j=1}^{s} a_{i,j} x_j \geq A \tag{11}$$

Goal functions for l^{th} priority level

$$\sum_{j=1}^{s} a_{i,j}^{l} x_j + n_{iu}^{l} - p_{iu}^{l} = T_{iu}^{l}, \text{ for } i = 1,2,...,r^{l} \tag{12}$$

Fuzzy Meta-goal variant 1 for l^{th} priority level

$$\sum_{i=1}^{r^l} \sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}} \lesssim Q_1^{l} \tag{13}$$

The membership function for FMG1 is defined as:

$$\mu\Big[\sum_{i=1}^{r^l} \sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}}\Big] = \begin{cases} 1 & ; \sum_{i \in L_u^{(1)}} \sum_{i=1}^{r^l} \sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}} \leq Q_1^{l} \\ 1 - \dfrac{\sum_{i=1}^{r^l} \sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}} - Q_1^{l}}{v_{1\max}^{(l)}} & ; Q_1^{l} \leq \sum_{i=1}^{r^l} \sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}} \leq Q_1^{l} + v_{1\max}^{(l)} \\ 0 & ; \sum_{i=1}^{r^l} \sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}} \geq Q_1^{l} + v_{1\max}^{(l)} \end{cases} \tag{14}$$

Fuzzy Meta-goal variant 2 for l^{th} priority level

$$\sum_{u=1}^{h} w_{iu}^{l} \frac{d_{iu}^{l}}{T_{iu}^{l}} - D^{l} \leq 0, \text{ for } i = 1,...,r^{l} \tag{15}$$

$$D^{l} \lesssim Q_2^{l} \tag{16}$$

For FMG2, the membership function is defined as:

$$\mu[D^{l}] = \begin{cases} 1 & ; D^{l} \leq Q_2^{l} \\ 1 - \dfrac{D^{l} - Q_v^{(2)}}{v_{2\max}^{(l)}} & ; Q_2^{l} \leq D^{l} \leq Q_2^{l} + v_{2\max}^{(l)} \\ 0 & ; D^{l} \geq Q_2^{l} + v_{2\max}^{(l)} \end{cases} \tag{17}$$

Meta-goal variant 3 for l^{th} priority level

$$-M_i^l < \sum_{u=1}^{h} d_{iu}^l - M_i^l y_i^l \le 0, \text{ for } i = 1,...,r^l \tag{18}$$

$$\frac{\sum_{i=1}^{r^l} y_i^l}{r^l} \lesssim Q_3^l \tag{19}$$

$$\mu\left[\frac{\sum_{i=1}^{r^l} y_i^l}{r^l}\right] = \begin{cases} 1 & ; \frac{\sum_{i=1}^{r^l} y_i^l}{r^l} \le Q_3^l \\ 1 - \frac{\frac{\sum_{i=1}^{r^l} y_i^l}{r^l} - Q_3^l}{v_{3\max}^{(l)}} & ; Q_3^l \le \frac{\sum_{i=1}^{r^l} y_i^l}{r^l} \le Q_3^l + v_{3\max}^{(l)} \\ 0 & ; \frac{\sum_{i=1}^{r^l} y_i^l}{r^l} \ge Q_3^l + v_{3\max}^{(l)} \end{cases}$$

Goal functions belonging to all higher priority levels

$$\sum_{j=1}^{s} a_{i,j}^{\rho} x_j^{\rho} + n_{iu}^{\rho} - p_{iu}^{\rho} = T_{iu}^{\rho};$$

$$\text{for } i = 1,2,...,r^{\rho}; \text{for } \rho = 1,...,l-1; \text{ for } u = 1,...,h \tag{20}$$

Fuzzy Meta-goal variants 1 constraint established in higher priority levels

$$\sum_{i=1}^{r^{\rho}} \sum_{u=1}^{s} w_{iu}^{\rho} \frac{d_{iu}^{\rho}}{T_{iu}^{\rho}} - \beta_1^{\rho} \le Q_1^{\rho}, \tag{21}$$

$$\mu\left(\sum_{i=1}^{r^{\rho}} \sum_{u=1}^{s} w_{iu}^{\rho} \frac{d_{iu}^{\rho}}{T_{iu}^{\rho}}\right) + \frac{\beta_1^{\rho}}{\beta_{1\max}^{\rho}} = 1, \tag{22}$$

$$\mu_1^{\rho} = R_1^{\rho} \text{ for } \rho = 1,...,l-1 \text{ and if } 0 \le \mu_1^{\rho} \le 1 \tag{23}$$

Fuzzy Meta-goal variants 2 constraints established in higher priority levels

$$\sum_{u=1}^{h} w_{iu}^{\rho} \frac{d_{iu}^{\rho}}{T_{iu}^{\rho}} - D^{\rho} \le 0, \text{ for } i = 1,...,r^{\rho} \tag{24}$$

$$D^{\rho} - \beta_2^{\rho} \le Q_2^{\rho}, \tag{25}$$

$$\mu(D^{\rho}) + \frac{\beta_2^{\rho}}{\beta_{2\max}^{\rho}} = 1 \tag{26}$$

$$\mu_2^{\rho} = R_2^{\rho} \text{ for } \rho = 1,\ldots,l-1 \text{ and if } 0 \le \mu_2^{\rho} \le 1 \tag{27}$$

Fuzzy Meta-goal variants 3 constraints established in higher priority levels

$$-M_i^{\rho} < \sum_{u=1}^{s} d_{iu}^{\rho} - M_i^{\rho} y_i^{\rho} \le 0, \text{ for } i = 1,\ldots,r^{\rho} \tag{28}$$

$$\frac{\sum_{i=1}^{r^{\rho}} y_i^{\rho}}{r^{\rho}} - \beta_3^{\rho} \le Q_3^{\rho}, \tag{29}$$

$$\mu\left(\frac{\sum_{i=1}^{r^{\rho}} y_i^{\rho}}{r^{\rho}}\right) + \frac{\beta_3^{\rho}}{\beta_{3\max}^{\rho}} = 1 \tag{30}$$

$$\frac{\sum_{i=1}^{r^{\rho}} y_i^{\rho}}{r^{\rho}} + \alpha_3^{\rho} - \beta_3^{\rho} = Q_3^{\rho} \tag{31}$$

$$\mu_3^{\rho} = R_3^{\rho} \text{ for } \rho = 1,\ldots,l-1 \text{ and if } 0 \le \mu_3^{\rho} \le 1 \tag{32}$$

Variable constraints

$$x_j, n_{iu}^{l}, p_{iu}^{l}, \beta_{1,2,3}^{l}, \beta_{1,2,3}^{\rho} \ge 0$$

$$D^{l}, D^{\rho} \ge 0$$

$$0 \le \mu_{1,2,3}^{\rho}, w_{iu}^{l}, w_{iu}^{\rho} \le 1$$

$$y_i^{l}, y_i^{\rho} \in \{0,1\} \tag{33}$$

II. EXAMPLE

A furniture manufacturer produces five kinds of products, chair, bench, table, sofa, bed. The production of all products is done in two separate machine centers within the plant. Each chair requires 2 hrs in machine center 1 and 1 hr in machine center 2, each bench requires 1 hrs in machine center 1 and 2 hr in machine center 2, each table requires 1 hrs in machine center 1 and 3 hr in machine center 2, Each sofa requires 4 hrs in machine center 1 and 1 hr in machine center 2, Each chair requires 3 hrs in machine center 1 and 1 hr in machine center 2. The gross margin from the sale of a chair is Rs. 100, from bench Rs.500, from table Rs.50, from sofa Rs.100 and from bed

Rs. 150. In addition, each product requires some in-process inventory. The per-unit in-process inventory required is Rs. 20, 20, 30, 30, 50 respectively for all five products.

The firm has normal monthly operation hours of 480 for machine center 1 and 400 for machine center 2. According to the marketing department, the forecasted sales for all five products is 150, 100, 120, 60, 10 units respectively for the coming month.

The plant manager has established the following goals for production in the next month:

1. Earn a gross profit of Rs.50000 in the next month.
2. Limit the amount tied up in in-process inventory for the month to Rs. 10,000.
3. Achieve the sale goal of all five products.
4. Avoid any underutilization of regular operation hours of both machine centers.
5. Limit the overtime operation to 100 hrs for each machine center 1 and 2.

Suppose the company regard a profit level of below Rs. 40,000 as twice as serious, Rs. 25,000 as four times as serious and below Rs. 20,000 as unacceptable. Also, the company decides that the amount tied up in in-process inventory for a month can lie between Rs. 10,000 to Rs. 10,500. An overall per unit penalty of 20% is applicable if it goes beyond Rs. 10,500.

Assuming the three groups are of equal importance, the following weighted goal programme is formulated:

III. FORMULATION

Let

x_i = Number of units of product to be sold; i=1, 2, 3, 4, 5

n_i = Underachievement of sales goal of product i

p_i = Overachievement of sales goal of product i

n_6 = Underutilization of regular operation hours of machine center 1

p_6 = Overachievement of regular operation hours of machine center 1

n_7 = Underutilization of regular operation hours of machine center 2

p_7 = Overachievement of regular operation hours of machine center 2

n_8^1 = Underachievement of amount tied up in in-process inventory for the month

p_8^1 = Overachievement of amount tied up in in-process inventory for the month

n_8^2 = Underachievement of increased amount tied up in in-process inventory for the month

p_8^2 = Overachievement of increased amount tied up in in-process inventory for the month

n_9^1 = Underachievement of profit level tied upto Rs. 50,000

p_9^1 = Overachievement of profit level tied upto Rs. 50,000

n_9^2 = Underachievement of second targeted profit level tied upto Rs. 40,000

p_9^2 = Overachievement of second targeted profit level tied upto Rs. 40,000

n_9^3 = Underachievement of third targeted profit level tied upto Rs. 25,000

p_9^3 = Overachievement of third targeted profit level tied upto Rs. 25,000

n_{10} = Underutilization of overtime operation hours of machine center 1

p_{10} = Overachievement of overtime operation hours of machine center 1

n_{11} = Underutilization of overtime operation hours of machine center 2

p_{11} = Overachievement of overtime operation hours of machine center 2

Thus, Weighted Goal Programming Problem is defined as:

Minimize: $n_1/150 + n_2/100 + n_3/120 + n_4/60 + n_5/10 + n_6/480 + n_7/400 + (p_8^1+0.2* p_8^2)/10000 + (n_9^1+ n_9^2+2* n_9^3)/50000 + p_{10}/100 + p_{11}/100$

Subject to

$x_1 + n_1 - p_1 = 150$

$x_2 + n_2 - p_2 = 100$

$x_3 + n_3 - p_3 = 120$

$x_4 + n_4 - p_4 = 60$

$x_5 + n_5 - p_5 = 10$

$2x_1 + x_2 + x_3 + 4 x_4 + 3 x_5 + n_6 - p_6 = 480$

$x_1 + 2 x_2 + 3x_3 + x_4 + x_5 + n_7 - p_7 = 400$

$20x_1 + 20 x_2 + 30 x_3 + 30 x_4 + 50 x_5 + n_8^1 - p_8^1 = 10000$

$20x_1 + 20 x_2 + 30 x_3 + 30 x_4 + 50 x_5 + n_8^2 - p_8^2 = 10500$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^1 - p_9^1 = 50000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^2 - p_9^2 = 40000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^3 - p_9^3 = 25000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 \geq 20000$

$p_6 + n_{10} - p_{10} = 100$

$p_7 + n_{11} - p_{11} = 100$

$x_i, n_j, p_j \geq 0$ (i = 1, 2, 3, 4, 5 ; j = 1, 2, . . ., 11)

Using the goal programming methodology, the solution is:

$x_1 = 80$, $x_2 = 100$, $x_3 = 50$, $x_4 = 60$, $x_5 = 10$,$n_1 = 70$, $p_1 = 0$, $n_2 = 0$, $p_2 = 0$,

$n_3 = 70$, $p_3 = 0$, $n_4 = 0$, $p_4 = 0$, $n_5 = 0$, $p_5 = 0$, $n_6 = 0$, $p_6 = 100$, $n_7 = 0$,

$p_7 = 100$, $n_8^1 = 2600$, $p_8^1 = 0$, $n_8^2 = 3100$, $p_8^2 = 0$, $n_9^1 = 0$, $p_9^1 = 18000$,

$n_9^2 = 0$, $p_9^2 = 28000$, $n_9^3 = 0$, $p_9^3 = 43000$,

$n_{10} = 0$, $p_{10} = 0$, $n_{11} = 0$, $p_{11} = 0$.

Let us assume that the decision-maker does not consider acceptable the above solution. Suppose the manager wants to set certain achievement level of the goals. Therefore, the model for calculating the payoff matrix is as follows, where three problems are solved for

$c_1 = 1, c_2 = 0, c_3 = 0; c_1 = 0, c_2 = 1, c_3 = 0;$

$c_1 = 0, c_2 = 0, c_3 = 1.$

Minimize: $c_1(n_1/150 + n_2/100 + n_3/120 + n_4/60 + n_5/10 + n_6/480 + n_7/400 + (p_8^1 + 0.2^* p_8^2)/10000 + (n_9^1 + n_9^2 + 2^* n_9^3)/50000 + p_{10}/100 + p_{11}/100) + c_2D + c_3(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11})/11)$

Subject to

$x_1 + n_1 - p_1 = 150$

$x_2 + n_2 - p_2 = 100$

$x_3 + n_3 - p_3 = 120$

$x_4 + n_4 - p_4 = 60$

$x_5 + n_5 - p_5 = 10$

$2x_1 + x_2 + x_3 + 4 x_4 + 3 x_5 + n_6 - p_6 = 480$

$x_1 + 2 x_2 + 3 x_3 + x_4 + x_5 + n_7 - p_7 = 400$

$20x_1 + 20 x_2 + 30 x_3 + 30 x_4 + 50 x_5 + n_8^1 - p_8^1 = 10000$

$20x_1 + 20 x_2 + 30 x_3 + 30 x_4 + 50 x_5 + n_8^2 - p_8^2 = 10500$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^1 - p_9^1 = 50000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^2 - p_9^2 = 40000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^3 - p_9^3 = 25000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 \geq 20000$

$p_6 + n_{10} - p_{10} = 100$

$p_7 + n_{11} - p_{11} = 100$

$z_1 - (n_1/150 + n_2/100 + n_3/120 + n_4/60 + n_5/10 + n_6/480 + n_7/400 + (p_8^1 + 0.2^*$

$p_8^2)/10000 + (n_9^1 + n_9^2 + 2^* n_9^3)/50000 + p_{10}/100 + p_{11}/100) = 0$

$-z_2 + D = 0$

$n_1 - 150D \leq 0$

$n_2 - 100D \leq 0$

$n_3 - 120D \leq 0$

$$n_4 - 60D \le 0$$
$$n_5 - 10D \le 0$$
$$n_6 - 360D \le 0$$
$$n_7 - 240D \le 0$$
$$p_8^{\ 1} + 0.2^* p_8^{\ 2} - 10000D \le 0$$
$$n_9^{\ 1} + n_9^{\ 2} + 2^* n_9^{\ 3} - 50000D \le 0$$
$$p_{10} - 100D \le 0$$
$$p_{11} - 100D \le 0$$
$$n_1 - 1500\, y_1 \le 0$$
$$n_2 - 1000\, y_2 \le 0$$
$$n_3 - 1200\, y_3 \le 0$$
$$n_4 - 600\, y_4 \le 0$$
$$n_5 - 100\, y_5 \le 0$$
$$n_6 - 3600\, y_6 \le 0$$
$$n_7 - 2400\, y_7 \le 0$$
$$(p_8^1 + 0.2^* p_8^2) - 100000\, y_8 \le 0$$
$$(n_9^1 + n_9^2 + 2^* n_9^3) - 500000\, y_9 \le 0$$
$$p_{10} - 1000\, y_{10} \le 0$$
$$p_{11} - 1000\, y_{11} \le 0$$
$$-z_3 + (y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11}) = 0;$$
$$x_i, n_j, p_j \ge 0 \ (i = 1, 2, 3, 4, 5;\ j = 1, 2, \ldots, 11)$$
$$y_i \in \{0,1\}$$

Where, $y_i = \begin{cases} 1, \text{ if goal i is not satisfied} \\ 0, \quad\quad\quad\quad \text{otherwise} \end{cases}$, i = 1, 2, ..., 11.

Three unique solutions will be obtained as the model configures to give strict preference to meta-goal variant 1 achievement ($c_1 = 1$, $c_2 = 0$, $c_3 = 0$), meta-goal variant 2 achievement ($c_1 = 0$, $c_2 = 1$, $c_3 = 0$), and meta-goal variant 3 achievement ($c_1 = 0$, $c_2 = 0$, $c_3 = 1$), respectively.

Thus the payoff matrix is as follow:

TABLE I

Objective						Achieved score		
	x_1	x_2	x_3	x_4	x_5	Aggregate unachievement	Maximum deviation	Number of unsatisfied goals
Aggregate unachievement	80	100	50	60	10	1.05	0.58	11
Maximum deviation	102	68	82	41	7	2.37	0.32	11
Number of unsatisfied goals	80	100	50	60	10	1.05	0.58	2

Based on this result, in order to analyze the decision problem further, the decision-makers nominate a fuzzy target value for each of the meta-goal variant considered. Thus the above weighted goal programme is extended to a fuzzy meta-goal programme with the following three fuzzy meta-goals:

FMG1: The percentage maximum deviation from all goals should lie between 1.05 to 1.5

FMG2: The maximum percentage deviation from any goal should lie between 0.3 to 0.6

FMG3: The number of unsatisfied goals should range from 2 to 4.

Thus, the Fuzzy Meta GP formulation is given as:

Maximize: $\mu_1 + \mu_2 + \mu_3$

Subject to

$$x_1 + n_1 - p_1 = 150$$

$$x_2 + n_2 - p_2 = 100$$

$$x_3 + n_3 - p_3 = 120$$

$$x_4 + n_4 - p_4 = 60$$

$$x_5 + n_5 - p_5 = 10$$

$$2x_1 + x_2 + x_3 + 4\,x_4 + 3\,x_5 + n_6 - p_6 = 480$$

$$x_1 + 2\,x_2 + 3\,x_3 + x_4 + x_5 + n_7 - p_7 = 400$$

$$20x_1 + 20\,x_2 + 30\,x_3 + 30\,x_4 + 50\,x_5 + n_8^1 - p_8^1 = 10000$$

$$20x_1 + 20\,x_2 + 30\,x_3 + 30\,x_4 + 50\,x_5 + n_8^2 - p_8^2 = 10500$$

$$100x_1 + 500\,x_2 + 50\,x_3 + 100\,x_4 + 150\,x_5 + n_9^1 - p_9^1 = 50000$$

$$100x_1 + 500\,x_2 + 50\,x_3 + 100\,x_4 + 150\,x_5 + n_9^2 - p_9^2 = 40000$$

$100x_1 + 500\ x_2 + 50\ x_3 + 100\ x_4 + 150\ x_5 + n_9^3 - p_9^3 = 25000$

$100x_1 + 500\ x_2 + 50\ x_3 + 100\ x_4 + 150\ x_5 \geq 20000$

$p_6 + n_{10} - p_{10} = 100$

$p_7 + n_{11} - p_{11} = 100$

$z_1 - (n_1/150 + n_2/100 + n_3/120 + n_4/60 + n_5/10 + n_6/480 + n_7/400 + (p_8^1 + 0.2^* p_8^2)/10000 + (n_9^1 + n_9^2 + 2^* n_9^3)/50000 + p_{10}/100 + p_{11}/100) = 0$

$-z_2 + D = 0$

$n_1 - 150D \leq 0$

$n_2 - 100D \leq 0$

$n_3 - 120D \leq 0$

$n_4 - 60D \leq 0$

$n_5 - 10D \leq 0$

$n_6 - 360D \leq 0$

$n_7 - 240D \leq 0$

$p_8^1 + 0.2^* p_8^2 - 10000D \leq 0$

$n_9^1 + n_9^2 + 2^* n_9^3 - 50000D \leq 0$

$p_{10} - 100D \leq 0$

$p_{11} - 100D \leq 0$

$n_1 - 1500\ y_1 \leq 0$

$n_2 - 1000\ y_2 \leq 0$

$n_3 - 1200\ y_3 \leq 0$

$n_4 - 600\ y_4 \leq 0$

$n_5 - 100\ y_5 \leq 0$

$n_6 - 3600\ y_6 \leq 0$

$n_7 - 2400\ y_7 \leq 0$

$(p_8^1 + 0.2^* p_8^2) - 100000\ y_8 \leq 0$

$(n_9^1 + n_9^2 + 2^* n_9^3) - 500000\ y_9 \leq 0$

$p_{10} - 1000\ y_{10} \leq 0$

$p_{11} - 1000\ y_{11} \leq 0$

$-z_3 + (y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11}) = 0;$

$z_1 - \beta_1 \le 1.05$

$$\mu_1 + \frac{\beta_1}{0.45} = 1$$

$z_2 - \beta_2 \le 0.3$

$$\mu_2 + \frac{\beta_2}{0.3} = 1$$

$$\frac{z_3}{11} - \beta_3 \le \frac{2}{11}$$

$$\mu_3 + \frac{\beta_3}{2} = 1$$

$$\mu_1 = 1 - \frac{z_1 - 1.05}{0.45}$$

$$\mu_2 = 1 - \frac{z_2 - 0.3}{0.3}$$

$$\mu_3 = 1 - \frac{\frac{z_3}{11} - \frac{2}{11}}{2}$$

$x_i, n_j, p_j, \beta_k \ge 0$ (i = 1, 2, 3, 4, 5 ; j = 1, 2, . . ., 11; k= 1, 2, 3)

$0 \le \mu_l \le 1$ (l = 1, 2, 3)

$y_i \in \{0,1\}$

Where, $y_i = \begin{cases} 1, \text{ if goal i is not satisfied} \\ 0, \quad \text{otherwise} \end{cases}$, i = 1, 2, ..., 11.

The solution for the above meta-goal programming problem is the following:

$x_1 = 86.36$, $x_2 = 68.18$, $x_3 = 69.09$, $x_4 = 60$, $x_5 = 10$, $n_1 = 63.63$, $p_1 = 0$,

$n_2 = 31.82$, $p_2 = 0$, $n_3 = 50.91$, $p_3 = 0$, $n_4 = 0$, $p_4 = 0$, $n_5 = 0$, $p_5 = 0$,

$n_6 = 0$, $p_6 = 100$, $n_7 = 0$, $p_7 = 100$, $n_8^1 = 2536.6$, $p_8^1 = 0$, $n_8^2 = 3036.36$,

$p_8^2 = 0$, $n_9^1 = 0$, $p_9^1 = 3681.818$, $n_9^2 = 0$, $p_9^2 = 13681.82$,

$n_9^3 = 0$, $p_9^3 = 28681.82$, $n_{10} = 0$, $p_{10} = 0$, $n_{11} = 0$, $p_{11} = 0$, $\mu_1 = 0.741$,

$\mu_2 = 0.586$, $\mu_3 = 0.95$, $b_1 = 0.12$, $b_2 = 0.124$, $b_3 = 0.091$.

All the fuzzy meta goals are not fully satisfied but partially with FMGs 1 and 3 achieved most. The aggregate deviation from all the goals is 1.16, maximum deviation from any goal is 0.42 and total number of unsatisfied goals is 3 out of 11. Let us now suppose that the DM establishes the following fuzzy meta-goals and preemptive priority levels in the second iteration:

Priority level 1: Satisfy goal 9 with the tolerance limit of 45000

Priority level 2: The aggregate deviation for goals 1, 2, 4, 5, 6 and 7 (with equal weights) must lie between 0.6 to 1.

Priority level 3: The maximum deviation for goals 3, 8, 10, 11 (with equal weights) must lie between 0.35 to 0.58.

Therefore, the following three-stage procedure is carried out:

Priority level 1:

Maximize: μ_1

Subject to

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 + n_9^1 - p_9^1 = 50000$

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 + n_9^2 - p_9^2 = 40000$

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 + n_9^3 - p_9^3 = 25000$

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 \geq 20000$

$z_1 = n_9^1 + n_9^2 + 2^* n_9^3$

$z_1 + \alpha_1 \geq 50000$

$\mu_1 + \alpha_1/5000 = 1$

$$\mu_1 = 1 - \frac{50000 - z_1}{5000}$$

$0 \leq \mu_1 \leq 1$

$x_i, n_j, p_j, \geq 0 \; (i = 1, 2, 3, 4, 5 \,;\, j = 9)$

The solution for the above problem is the following:

$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 150,$

$n_9^1 = 27500, p_9^1 = 0, n_9^2 = 17500, p_9^2 = 0,$

$n_9^3 = 2500, p_9^3 = 0, \alpha_1 = 0, \mu_1 = 1, z_1 = 50000,$

meeting the first fuzzy meta-goal.

Priority level 2:

Maximize: μ_2

Subject to

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 + n_9^1 - p_9^1 = 50000$

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 + n_9^2 - p_9^2 = 40000$

$100x_1 + 500\, x_2 + 50\, x_3 + 100\, x_4 + 150\, x_5 + n_9^3 - p_9^3 = 25000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 \geq 20000$

$z_1 = n_9^1 + n_9^2 + 2^* n_9^3$

$z_1 + \alpha_1 \geq 50000$

$\mu_1 + \alpha_1/5000 = 1$

$$\mu_1 = 1 - \frac{50000 - z_1}{5000}$$

$\mu_1 = 1$

$x_1 + n_1 - p_1 = 150$

$x_2 + n_2 - p_2 = 100$

$x_4 + n_4 - p_4 = 60$

$x_5 + n_5 - p_5 = 10$

$2x_1 + x_2 + x_3 + 4 x_4 + 3 x_5 + n_6 - p_6 = 480$

$x_1 + 2 x_2 + 3 x_3 + x_4 + x_5 + n_7 - p_7 = 400$

$z_2 - (n_1/150 + n_2/100 + n_4/60 + n_5/10 + n_6/480 + n_7/400) = 0$

$z_2 - \beta_2 \leq 0.6$

$$\mu_2 + \frac{\beta_2}{0.4} = 1$$

$$\mu_2 = 1 - \frac{z_2 - 0.6}{0.4}$$

$0 \leq \mu_2 \leq 1$

$x_i, n_j, p_j, \beta_k \geq 0$ (i = 1, 2, 3, 4, 5 ;j = 1, 2, 3, 4, 5, 9; k = 1)

The solution for the above problem is the following:

$x_1 = 150$, $x_2 = 40$, $x_3 = 33.3$, $x_4 = 60$, $x_5 = 10$, $n_1 = 0$, $p_1 = 0$,

$n_2 = 60$, $p_2 = 0$, $n_4 = 0$, $p_4 = 0$, $n_5 = 0$, $p_5 = 0$, $n_6 = 0$, $p_6 = 163.3$,

$n_7 = 0$, $p_7 = 0$, $n_9^1 = 5833.3$, $p_9^1 = 0$, $n_9^2 = 0$, $p_9^2 = 4166.6$,

$n_9^3 = 22083.33$, $p_9^3 = 41250$, $\alpha_1 = 0$, $\beta_2 = 0$, $z_1 = 50000$, $z_2 = 0.6$, $\mu_1 = 1$, $\mu_2 = 1$

meeting the second fuzzy meta-goal.

Priority level 3:

Maximize: μ_3

Subject to

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^1 - p_9^1 = 50000$

$100x_1 + 500 x_2 + 50 x_3 + 100 x_4 + 150 x_5 + n_9^2 - p_9^2 = 40000$

$100x_1 + 500\,x_2 + 50\,x_3 + 100\,x_4 + 150\,x_5 + n_9^3 - p_9^3 = 25000$

$100x_1 + 500\,x_2 + 50\,x_3 + 100\,x_4 + 150\,x_5 \geq 20000$

$z_1 = n_9^1 + n_9^2 + 2^* n_9^3$

$z_1 + \alpha_1 \geq 50000$

$\mu_1 + \alpha_1/5000 = 1$

$$\mu_1 = 1 - \frac{50000 - z_1}{5000}$$

$\mu_1 = 1$

$x_1 + n_1 - p_1 = 150$

$x_2 + n_2 - p_2 = 100$

$x_4 + n_4 - p_4 = 60$

$x_5 + n_5 - p_5 = 10$

$2x_1 + x_2 + x_3 + 4\,x_4 + 3\,x_5 + n_6 - p_6 = 480$

$x_1 + 2\,x_2 + 3\,x_3 + x_4 + x_5 + n_7 - p_7 = 400$

$z_2 - (n_1/150 + n_2/100 + n_4/60 + n_5/10 + n_6/480 + n_7/400) = 0$

$z_2 - \beta_2 \leq 0.6$

$$\mu_2 + \frac{\beta_2}{0.4} = 1$$

$$\mu_2 = 1 - \frac{z_2 - 0.6}{0.4}$$

$20x_1 + 20\,x_2 + 30\,x_3 + 30\,x_4 + 50\,x_5 + n_8^1 - p_8^1 = 10000$

$20x_1 + 20\,x_2 + 30\,x_3 + 30\,x_4 + 50\,x_5 + n_8^2 - p_8^2 = 10500$

$x_3 + n_3 - p_3 = 120$

$p_6 + n_{10} - p_{10} = 100$

$p_7 + n_{11} - p_{11} = 100$

$n_3 - 120D \leq 0$

$p_8^1 + 0.2^* p_8^2 - 10000D \leq 0$

$p_{10} - 100D \leq 0$

$p_{11} - 100D \leq 0$

$-z_3 + D = 0$

$Z_3 - \beta_3 \leq 0.35$

$$\mu_3 + \frac{\beta_3}{0.18} = 1$$

$$\mu_3 = 1 - \frac{z_3 - 0.35}{0.18}$$

$0 \le \mu_l \le 1$ $(l = 1, 2)$

$x_i, n_j, p_j, \alpha_k, \beta_k \ge 0$ ($i = 1, 2, 3, 4, 5; j = 1, \ldots, 11; k = 1, 2, 3$).

The solution for the above problem is the following:

$x_1 = 100.78$, $x_2 = 72.82$, $x_3 = 73.98$, $x_4 = 60$, $x_5 = 10$,

$n_1 = 49.2$, $p_1 = 0$, $n_2 = 27.18$, $p_2 = 0$, $n_3 = 46.02$, $p_3 = 0$, $n_4 = 0$, $p_4 = 0$,

$n_5 = 0$, $p_5 = 0$, $n_6 = 0$, $p_6 = 138.35$, $n_7 = 0$, $p_7 = 138.35$,

$n_8^1 = 2008.7$, $p_8^1 = 0$, $n_8^2 = 2508.74$, $p_8^2 = 0$, $n_9^1 = 0$, $p_9^1 = 7684.47$,

$n_9^2 = 0$, $p_9^2 = 17684.47$, $n_9^3 = 25000$, $p_9^3 = 57684.47$, $n_{10} = 0$, $p_{10} = 38.35$,

$n_{11} = 0$, $p_{11} = 38.35$, $\alpha_1 = 0$, $\beta_2 = 0$, $\beta_3 = 0.033$, $z_1 = 50000$, $z_2 = 0.6$,

$z_3 = 0.383$, $\mu_1 = 1$, $\mu_2 = 1$, $\mu_3 = 0.81$.

Thus, all the three fuzzy meta-goals are satisfied. The table II provides the solution:

TABLE II

x_1	x_2		x_3	x_4	x_5	
101	73		74	60	10	
	Goal type		Current achievement level	Target value of the goal	Status of achievement	
Meta goal	Level 1	1	0		Achieved	
	Level 2	1	0.6	0.6 - 1	Achieved	
	Level 3	2	0.35	0.35-0.58	Achieved	
		1	101	≥ 150	Not achieved	
		2	73	≥ 100	Not achieved	
		3	74	≥ 120	Not achieved	
Decision goal solution		4	60	≥ 60	Achieved	

	5	10	≥ 10	Achieved
	6	619	≥ 480	Achieved
	7	539	≥ 400	Achieved
	8	8000	≤ 10000	Achieved
	9	27800	≥ 50000	Achieved
	10	138.35	≤ 100	Over Achieved
	11	138.35	≤ 100	Over Achieved

Goal 9 of profit maximization is fully achieved. Here, machine center 1 and 2 are over achieved. Amount spent in in-process inventory is within the target limit.

So, based on manager's priorities, the achievement level of different goals can be attained.

IV. CONCLUSION

This paper uses IFMGP-based decision-analysis method. Interactive method is sensitivity analysis on the target values where the decision-maker first propose goals for the problem and then the goals are cross-verified and analyzed to obtain the final solution. Further, the target values of the goals are considered imprecise in nature and then the results are verified. Thus, IFMGP can help to solve two main problems associated to the use of GP as an decision-making tool: the allocation of target levels to each attribute and the selection of a suitable achievement function assuming fuzziness in the target values. Further research can be done by applying weights and normalizing factors for fuzzy meta-goals.

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