*International Journal of Operations Research and Optimization*
January-June 2017, Volume 8, No. 1,*pp-12-22*
*ISSN-(Print)0975-3737, (Online)2231-4741*


# PREDICTING THE GROWTH OF MANPOWER SYSTEM USING MARKOV CHAIN APPROACH

**R. Elangovan* and M. Govindhan**
Department of Statistics
Annamalai University, Annamalai Nagar – 608 002
Tamilnadu, India
Email: srelangovan@rediffmail.com

**ARTICLE INFO**

**Article history :**
Received:15-01-2017
Accepted :20-02-2017

**Keywords Used:**
Stability and Maintainable Structures, Graded Manpower Systems, Manpower Attrition, Stochastic Approach

**ABSTRACT**

The concept of stability and maintainable structures in a graded manpower systems is of central importance in theoretical and applied study of populations. Predicting the manpower attrition using Stochastic approach has received much attention in recent literature. Stochastic model for the description, prediction and control of attrition from the various grades of a hierarchically structured manpower system of management staff is discussed by Vassiliou (1976). In this paper, it is proposed to study the manpower attrition using Stochastic approach in a global perspective. This approach is highly useful for not only predicting the human manpower but also formulating human resource strategy of Embryonic India.

*Presented in the VIII International Symposium on Statistics and Optimization in Conjunction with XXXVI-Annual Convention of Indian Society for Probability & Statistics (ISSAC-2016), 17-19, December 2016, India.

## 1. INTRODUCTION

The concept of stability maintainable structures in a graded manpower systems is of central importance in theoretical and applied study of populations. Predicting the growth of manpower system using Stochastic approach has received much attention in recent literature. Stochastic models for the description, prediction and control of wastage in hierarchically structured manpower systems is discussed by many authors in the recent past. Stochastic model for the description, prediction and control of wastage from the various grades of a hierarchically structured manpower system of management staff is discussed by Vassiliou (1976). In this paper, it is proposed to study the manpower attrition using Stochastic approach in a global perspective. This approach is highly useful for not only predicting the human manpower but also formulating human resource strategy of Embryonic India

## 2. METHODOLOGY AND DATA COLLECTION

The employees in the software industries were characterized as being in the following categories grade 1 (manager), grade 2 (senior manager), grade 3 (technical analyst), grade 4 (senior analyst), grade 5 (junior engineer), Grade 6 (associate engineer), grade 7 (console operator). A detailed manpower analysis based on the grades recruitment, promotion and withdrawal was carried out based on the grades using Stochastic approach. The manpower model was then used to predict manpower stocks and flows of the industry and various growth scenarios. A detailed survey of a representative sample of these firms is then carried out and used to estimate recruitment, transfer (within the industry), promotion and leaving rates. The statistical analysis was carried out using SPSS package and solving the TPM matrix using MATLAB package. Some interesting results can also be seen in McClean (1980), Zanakis and Maret (1980), Vassiliou (1981), Davies (1982), Bartholomew (1982), Guerry (1991), McClean et. al. (1991).

## 2. MARKOV MODELS

The model developed here is a transition model based on Markov chians, as described by Bartolomew et al. (1991) by considering seven grades A discrete time Markov chain is a Markov process whose state space is a finite or countable set, whose time set is T= (0,1,2,...) from the Markov property

$$\Pr\{X_{n+1} = j / X_n = i\} \quad \text{... (1)}$$

The probability of the system $X_{n+1}$ being in state j given that $X_n$ is in state i is called one step transition probability and it is denoted by $p_{ij}^{n,n+1}$

$$p_{ij}^{n,n+1} = \Pr\{X_{n+1} = j / X_n = i\} \text{, i = 1, 2, 3, 4, 5,6,7} \quad \text{... (2)}$$

**3. NOTATIONS**

$N(t) = \sum_{i-1}^{7} n_i(t)$ the total size of staff at the beginning of the $t^{th}$ session

$n_i(t)$ = Number of staff in cader i at the beginning of the $t^{th}$ session

$n_{ij}(t)$ = *Number of the person who move from grade i to j at $t^{th}$ session*

$n_{ij}$ = Wastage flow from $i^{th}$ cadre within the $t^{th}$ session.

$n_{0j}(t)$ = The recrutiment flow from j at the beginning of the $t^{th}$ session

$p_{ij}(t)$ = The transition probaility of a person in grade i moving to grade j within the $t^{th}$ session.

where i , j = 1,2,3...,7: 'states of the system, representing the various grade of levels of members of staff of the organization;

**4. ASSUMPTIONS**

$$\sum_{j=1}^{7} p_{0j} = 1 \quad \text{... (3)}$$

Wastage and promotion are intimately connected.

$$\sum_{j=1}^{7} p_{ij} + w_i = 1 \quad \text{... (4)}$$

$$w_i = 1 - \sum_{i=1}^{m} p_{ij} \quad \text{... ( 5)}$$

**5. DETERMINATION OF TRANSITION PROBABILITIES**

The system is divided into k categories will described as grades. The transition probabilities between each of the grades are set out in an array form

$$\begin{bmatrix} p_{11} & p_{22}\cdots & p_{1k} & w_1 \\ p_{21} & p_{22}\cdots & p_{2k} & w_2 \\ \cdots\cdots & \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots\cdot \\ p_{k1} & p_{k2} & p_{kk} & w_k \end{bmatrix}$$

The probabilities $p_{ij}(t)$ gives the estimates of $p_{ij}$ as

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i} \quad i = 12,...7, \; j = 1,2,...7 \quad \text{... (6)}$$

$$\widehat{p}_{ij} = \frac{\sum_{i=1}^{6} n_{ij}(t)}{\sum_{i=1}^{6} n_i(t)} \quad , i, j = 12,...7.$$

... (7)

## 6. STATIONARY OF TRANSITION PROBABILITIES

Assumption of constant transition probabilities over time implies that

$$p_{ij}(t) = p_{ij} \qquad for all\ j = 1,2,...7$$

Testing hypothesis is stated as:

H0 : Transition Probabilities are constant overtime

H1 : Transition Probabilities are not constant overtime

To test stationary of the seasonal of the seasonal TPM's pi with elements $\widehat{p}_{ij}(t)$ we use the following layout below. i = 1,2,..7.

The χ2 test of stationary specify that Transitions from row state i to state j are stationary at α- level of significance if

$$\chi_i^{\,2} = \sum_{j(i)=1}^{7} \sum_{t=1}^{6} n_i(t) \frac{\left[p_{ij}(t) - p_{ij}\right]^2}{p_{ij}} < \chi^2_{(\alpha, 2(m-1)}$$

Where m is the number of pij 's > 0 The entire TPM, P is constant overtime if

$$\chi_i^{\,2} = \sum_{i=1}^{7} \sum_{j(i)=1}^{7} \sum_{t=1}^{6} n_i(t) \frac{\left[p_{ij}(t) - p_{ij}\right]^2}{p_{ij}} < \chi^2_{(\alpha, 2(m-1)}$$

Where m is the number of pij 's > 0

## 7. THE PREDICTION EQUATION FOR EXPECTED STAFF STRUCTURE

Let $\underline{\bar{n}}(t) = (\bar{n}_1(t), \bar{n}_2(t), \bar{n}_3(t), ..., \bar{n}_7(t)$ be the vector of cadre sizes at the beginning of the tth session, where the top and bottom bar notation denote expectation and vector respectively. It can be shown that

$$\underline{\bar{n}}(t+1) = \bar{n}(t)Q$$

where $Q = P + W^T r \Rightarrow q_{ij} = p_{ij} + w_i r_j$

P = 7x7, over all transition probability matrix (TPM)

wT = 7x1, vector of wastage probabilities

r = 1x7, vector of average recruitment probabilities

**8. RESULTS**

The detailed manpower analysis of all recruitment, promotion and wastage was carried out based on the selected software companies with respect to recruitment, promotion, wastage and the results were shown in table 1, table 2.

**TABLE 1:**

**MANPOWER FLOWS IN SOFTWARE COMPANIES**

| Year | Manager (M) | | | Senior Manager (SM) | | | | Technical analyst (TA) | | | | Senior analyst (SA) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | R(M) | M | W (M) | R (SM) | SM | P (CM) | W (SM) | R (TA) | TA | P (SM) | W (SM) | R (SA) | SA | P (TA) | W (TA) |
| 2009 | 0 | 10 | 0 | 16 | 22 | 5 | 1 | 5 | 6 | 4 | 3 | 3 | 8 | 9 | 6 |
| 2010 | 3 | 15 | 0 | 12 | 24 | 10 | 6 | 5 | 10 | 5 | 5 | 7 | 12 | 6 | 4 |
| 2011 | 4 | 20 | 0 | 11 | 22 | 7 | 5 | 4 | 15 | 6 | 1 | 3 | 15 | 6 | 3 |
| 2012 | 2 | 15 | 0 | 7 | 19 | 5 | 4 | 5 | 12 | 3 | 5 | 4 | 17 | 3 | 2 |
| 2013 | 2 | 10 | 0 | 4 | 16 | 4 | 2 | 4 | 10 | 4 | 5 | 3 | 13 | 2 | 3 |
| 2014 | 0 | 11 | 1 | 5 | 14 | 4 | 3 | 5 | 12 | 2 | 4 | 5 | 12 | 4 | 3 |

Cont..,

| Year | Junior Engineer (JE) | | | | Associate Engineer (AE) | | | | Console Operator (CO) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | R(JE) | JE | P (JE) | W(JE) | R(AE) | AE | P(AE) | W(AE) | R(CO) | CO | P(CO) | W(CO) |
| 2009 | 5 | 12 | 4 | 0 | 5 | 12 | 6 | 4 | 3 | 5 | 3 | 0 |
| 2010 | 7 | 14 | 5 | 0 | 2 | 15 | 4 | 3 | 2 | 1 | 0 | 3 |
| 2011 | 5 | 11 | 3 | 2 | 4 | 10 | 4 | 2 | 5 | 4 | 3 | 2 |
| 2012 | 4 | 10 | 1 | 3 | 4 | 14 | 3 | 4 | 7 | 5 | 2 | 4 |
| 2013 | 5 | 13 | 5 | 4 | 5 | 12 | 3 | 5 | 4 | 3 | 4 | 2 |
| 2014 | 10 | 14 | 4 | 1 | 4 | 10- | 3 | 2 | 5 | 3 | 2 | 1 |

Manager (M), Senior Manager (SM), Technical analyst (TA), Senior analyst (SA),Junior Engineer (JE), Associate Engineer (AE), Console Operator. R-recruitment, P-Promotion, W-Wastage.

**TABLE 2:**

**MANPOWER FLOW IN VARIOUS GRADES IN SOFTWARE INDUSTRIES**

| T | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
|---|---|---|---|---|---|---|---|---|
| $N_{oj}(t)^i$ | | | | | | | | $n_i(t)$ |
| | 12(0.7000) | 8(0.3000) | | | | | | 20 |
| | 13(0.6538) | 13(0.3462) | | | | | | 26 |
| | 25(0.8182) | 8(0.1818) | | | | | | 33 |
| 1 | 15(0.8000) | 6(0.2000) | | | | | | 20 |
| | 20(0.8636) | 2(0.1364) | | | | | | 22 |
| | 11(0.8571) | 3(0.1429) | | | | | | 14 |
| | 105(0.9292) | 30(0.2655) | | | | | | 113 |
| | | 21(0.8571) | 7(0.0714) | | | | | 28 |
| | | 28(0.7501) | 8(0.1111) | | | | | 36 |
| | | 28(0.7188) | 4(0.1563) | | | | | 32 |
| 2 | | 24(0.8077) | 2(0.0769) | | | | | 26 |
| | | 17(0.8500) | 2(0.1000) | | | | | 20 |
| | | 16(0.8235) | 1(0.1765) | | | | | 17 |
| | | 126(0.7925) | 16(0.1006) | | | | | 159 |
| | | | 8(0.5789) | 6(0.4000) | | | | 15 |
| | | | 10(0.6316) | 4(0.2105) | | | | 19 |
| | | | 16(0.7619) | 3(0.1429) | | | | 21 |
| 3 | | | 13(0.7647) | 2(0.1176) | | | | 17 |
| | | | 12(0.7500) | 1(0.0625) | | | | 16 |
| | | | 14(0.7363) | 3(0.1579) | | | | 14 |
| | | | 75(0.7009) | 19(0.1776) | | | | 107 |
| | | | | 12 (0.5789) | 7(0.1579) | | | 19 |
| | | | | 13(0.6667) | 4(0.1905) | | | 21 |
| | | | | 18(0.8182) | 2(0.0909) | | | 22 |
| 4 | | | | 19(0.8261) | 3(0.1304) | | | 23 |
| | | | | 16(0.8000) | 2(0.1000) | | | 20 |
| | | | | 14(0.7368) | 3(0.1579) | | | 19 |
| | | | | 92(0.7419) | 17(0.1371) | | | 124 |

| | | | | | |
|---|---|---|---|---|---|
| | 13(0.7646) | 4(0.2353) | | | 17 |
| | 16(0.7619) | 5(0.2381) | | | 21 |
| | 12(0.7500) | 3(0.1875) | | | 16 |
| 5 | 11(0.7333) | 2(0.1333) | | | 15 |
| | 14(0.8235) | 2(0.1176) | | | 17 |
| | 17(0.8520) | 1(0.5000) | | | 20 |
| | 83(0.7830) | 17(0.1604) | | | 106 |
| | | 13(0.7222) | 2(0.1111) | | 18 |
| | | 16(0.8421) | 1(0.0526) | | 19 |
| | | 12(0.8020) | 2(0.1333) | | 15 |
| 6 | | 15(0.8333) | 1(0.5556) | | 18 |
| | | 13(0.7647) | 2(0.1176) | | 17 |
| | | 11(0.7648) | 4(0.1875) | | 16 |
| | | 81(0.7864) | 11(0.1068) | | 103 |
| | | | | 6(0.8571) | 7 |
| | | | | 4(0.6687) | 6 |
| | | | | 5(0.6250) | 8 |
| 7 | | | | 3(0.6000) | 5 |
| | | | | 2(0.6667) | 3 |
| | | | | 6(0.7500) | 8 |
| | | | | 26(0.7027) | 37 |

Key : $n_i(t)$ = Number of staff in cadre i at beginning of the t th session, $N_{oj}(t)^i$ = Total recruitment flow to grade j at the beginning of the tth session

**7. ANALYSIS AND RESULTS**

The data presented in table 8.1, the transition probability matrix for the seven grade are shown below

$$P=\begin{pmatrix} 0.0192 & 0.2177 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.6211 & 0.2175 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.5001 & 0.1665 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2332 & 0.1334 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6233 & 0.1604 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6223 & 0.2210 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1132 & 0.6143 \end{pmatrix}$$

**TABLE 3:**

**STATIONARITY OF THE TRANSITION PROBABILITIES**

| Cadre (i) | $\chi_i^{2}$ | Df | $\chi^2_{(0.05,df)}$ | p-value |
|---|---|---|---|---|
| 1 | 3.12 | 5 | 12.04 | 0.71 |
| 2 | 2.13 | 5 | 12.04 | 0.34 |
| 3 | 3.14 | 5 | 12.04 | 0.54 |
| 4 | 1.07 | 5 | 12.04 | 0.45 |
| 5 | 2.04 | 5 | 12.04 | 0.67 |
| 6 | 1.43 | 5 | 12.04 | 0.92 |
| 7 | 0.42 | 5 | 12.04 | 0.91 |
| Total | 12.97 | 35 | 52.14 | 0.91 |

The test Statistics for TPM is suggested by Ossai and Uche (2009)

$$\chi_i^{2} = \sum_{i=1}^{7} \sum_{j(i)=1}^{7} \sum_{t=1}^{6} n_i(t) \frac{\left[\mathrm{p_{ij}(t) - p_{ij}}\right]^2}{p_{ij}} = 12.271$$

The TPM are constant over time based on different cadre.

(i) Prediction of Future Structure

From the table 3 recruitment vector and wastage probabilities were calculated and are shown as in appendix

$$r = \begin{bmatrix} 0.05 & 0.33 & 0.23 & 0.13 & 0.33 & 0.26 & 0.52 \end{bmatrix}$$ and

$$w = \begin{bmatrix} 0.72 & 0.10 & 0.15 & 0.11 & 0.04 & 0.12 & 0.27 \end{bmatrix}$$

$$P = \begin{pmatrix} 0.021 & 0.2423 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7925 & 0.1112 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.5423 & 0.2335 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.6421 & 0.1231 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6543 & 0.1439 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6543 & 0.2133 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0324 \end{pmatrix}$$

As the computation of $Q = P + W^T r$ is

$$Q = \begin{pmatrix} 0.0718 & 0.4856 & 0.1562 & 0.1349 & 0.1123 & 0.2059 & 0.4047 \\ 0.0066 & 0.8266 & 0.1248 & 0.0209 & 0.0429 & 0.0319 & 0.0627 \\ 0.0072 & 0.0372 & 0.7273 & 0.2124 & 0.0468 & 0.0348 & 0.0542 \\ 0.0072 & 0.0372 & 0.0211 & 0.7647 & 0.1839 & 0.0348 & 0.0684 \\ 0.0036 & 0.0186 & 0.0132 & 0.0114 & 0.8064 & 0.1778 & 0.0342 \\ 0.0066 & 0.0341 & 0.0242 & 0.0209 & 0.0429 & 0.7423 & 0.1695 \\ 0.0174 & 0.0899 & 0.0638 & 0.0551 & 0.1131 & 0.0841 & 0.0341 \end{pmatrix}$$

Computations are carried out using a program in MATLAB and SPSS package and the manpower flow is shown in table (8. 4) using equ (8.9) and equ. (8.10) and prediction of staffing structure shown in Figure 1

**TABLE 4:**

**OBSERVED EXPECTED MANPOWER STRUCTURE**

| Year | T | 1 | 2 | 3 | 4 | 5 | 6 | 7 | N(t) |
|---|---|---|---|---|---|---|---|---|---|
| 2009 | 1 | 20 | 31 | 14 | 13 | 11 | 8 | 6 | 103 |
| 2010 | 2 | 18 | 30 | 15 | 13 | 10 | 7 | 4 | 105 |
| 2011 | 3 | 13 | 27 | 16 | 13 | 8 | 10 | 4 | 105 |
| 2012 | 4 | 17 | 26 | 16 | 18 | 10 | 9 | 5 | 105 |
| 2013 | 5 | 16 | 24 | 15 | 14 | 12 | 9 | 6 | 99 |
| 2014 | 6 | 18 | 24 | 15 | 12 | 11 | 7 | 4 | 91 |
| 2015 | 7 | 17 | 23 | 13 | 11 | 8 | 6 | 4 | 82 |
| 2016 | 8 | 17 | 23 | 14 | 10 | 8 | 4 | 5 | 81 |
| 2017 | 9 | 17 | 23 | 15 | 1 | 8 | 3 | 5 | 81 |

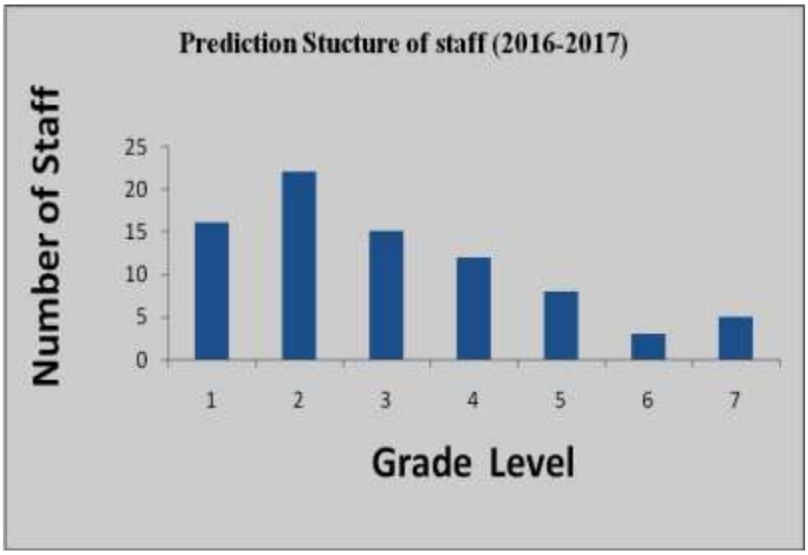


**Figure 1: Prediction Structures of staff for 2016-2017 session**

The result shows that the manpower flow based on TPM between one grade to another grade is Stationary, since, the p value = 0.92 then TPM are constant over time based on different cadre. Also from table 4 shows that the expected staffing structure of Chennai software companies presented in table 1 between different grade remain unchanged based on promotion, recruitment and attrition. It is evident that based on Stochastic chain approach the transition period is stationary over the predicting period and future prediction also possible based staffing structure with respect to recruitment, promotion and attrition. Future it is evident that the staffing structure remain unchanged based on stationary.

**8, CONCLUSION**

It is clear that , predicting the growth of manpower attrition using Stochastic approach, the transition period is stationary over the predicting period and future prediction also possible based staffing structure with respect to recruitment, promotion and attrition. Future, it is evident that the staffing structure remains unchanged based on stationary. This approach is highly useful for not only predicting the human manpower but also formulating human resource strategy of Embryonic India.

**REFERENCE**


1. McClean, S. I. (1980). A Semi-Markov Model for a Multi-Grade Population with Poisson Recruitment. Journal of Applied Probability, Vol. 17, pp.846-354.
2. McClean, S. I. (1991). Manpower Planning Models and their Estimation. European Journal of Operational Research, Vol. 51, pp. 179-187.
3. Vassiliou P. C. G. (1976). A Markov Chain Model for Wastage in Manpower Systems, Operational Research Quarterly, Vol.27, pp. 57-70.
4. Vassiliou, P. C. G. (1981). Stability in a Non-Homogeneous Markov Chain Model in Manpower Systems. Journal of Applied Probability, Vol.18, pp. 924-930.
5. Vassiliou, P. C. G. (1982). On the Limiting Behaviour of a Non-Homogeneous Markovian Manpower Model with Independent Poisson Input. Journal of Applied Probability, Vol. 19, No. 2, pp. 433-438.
6. Davies, G. (1982). Control of Grade Sizes in a Partially Stochastic Markova Manpower Model. Journal of Applied Probability, Vol.19, pp. 439-443.
7. Guerry, M. A. (1991). Monotonicity Property of t-Step Maintainable Structures in Three - Grade Manpower System: A Counter Example. Journal of Applied Probability, Vol. 28, pp. 221-224.