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Triply-Diffusive Magneto Convection in Viscoelastic Fluid Through Porous Medium

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ABSTARCT

The triply diffusive convection in a Maxwell viscoelastic fluid is mathematically investigated in the presence of uniform vertical magnetic field through porous medium. Using linearized stability theory and normal mode analysis, the dispersion relation is obtained. The magnetic field and solute gradients are found to have stabilizing effect, whereas medium permeability has destabilizing effect on the system for stationary convection. Graphs have been plotted by giving numerical values of the parameters to depict the stability characteristics. Further, solute gradients and magnetic field are found to introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Key Words- Triply-diffusive convection, Solute gradients,Maxwellian viscoelastic fluid, Magnetic field, Porous medium

Introduction

The theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagneticshas been treated in detail by Chandrasekhar [1] in his celebrated monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis[2]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirabletoconsider a fluid acted on by a solute gradient and free boundaries.

In the standard Beˈnard problem, the instability is driven by a density difference caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference, the temperature field and salt field. The solution behavior in the double-diffusive convection problem is more interesting that of the single component situation in so much as new instability phenomena may occur which is not present in the classical Beˈnard problem. Although the subject of double-diffusive convection is still an active research area, however, there are many fluid systems in which more than two components are present. For example, Degens et al. [3] reported that the saline waters of geothermally heated Lake Kivu strongly stratified by temperature and a salinity which is the sum of comparable concentrations of many salts, while the oceans contain many salts in concentrations less than a few per cent of the sodium chloride concentration. It has been recognized previously that there are important fluid mechanical systems in which the density depends on three or more stratifying agencies having different diffusivities, which can be called multiply diffusive convection [4]. By analogy with the doubly diffusive case in which the density depends on two independently diffusing stratifying agencies, we refer to the isothermal quaternary and non-isothermal ternary (i.e. three-component) cases as being 'triply-diffusive'. When temperature and two or more component agencies, or three different salts, are present then the physical and mathematical situation becomes increasingly richer. Very interesting results in triply diffusive convection have been obtained by Pearlstein et al. [4]. They demonstrate that for triple diffusive convection linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region in between the linear instability ones. This is because for certain parameters the neutral curve has a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one.

Research on fluid motions in porous media is an area of great activity today-as it was in the past-either because of its great geophysical relevance (engineering geology, subsurface and structural geology, subsurface fluid motions etc.) or because porous materials (like fiber materials used for insulating purposes or metallic foams in heat transfer devices) occur frequently and influence all of our lives. Rionero [5] studied a triply convective-diffusive fluid mixture saturating a porous horizontal layer in the Darcy-Oberbecle-Boussinesq scheme. Tracey [6] developed the linear instability and nonlinear energy stability analyses for the problem of a fluid-saturated porous layer stratified by penetrative thermal convection and two salt concentrations.

Recently, interest in viscoelastic flows through porous media has grown considerably, due largely to the demands of such diverse fields as biorheology, geophysics, chemical, and petroleum industries. Wang and Tan [7] have studied the stability analysis of double diffusive convection in Maxwell fluid in a porous medium. It is worthwhile to point out that the first viscoelastic rate type model, which is still used widely, is due to Maxwell. Kim et al. [8] have studied the thermal instability of viscoelastic fluids in porous media. Malashetty et al. [9] have studied the double diffusive convection in a binary viscoelastic fluid saturated anisotropic porous layer and found that the effect of the stress relaxation parameter is to advance the onset of oscillatory convection whereas the strain retardation parameter delaysthe onset of oscillatory convection.Wang and Tan [10] have discussed the stability of Soret-driven double-diffusive convection of Maxwell fluid in a porous medium while the SoretDufour driven thermosolutal instability of Darcy-Maxwell fluid is studied by Jaimala [11].

In the double-diffusive convection, fluid contains two components with different molecular diffusivities. But there are many situations where more than two components are involved like the solidification of molten alloys, geothermally heated lakes, magmas and their laboratory models and sea water. Rionero [12] has studied the triple diffusive convection in porous media. Keeping in mind the importance in various fields and in view of the recent increase in the number of non iso-thermal situations, we intend to perform linear stability analysis of a triply diffusive convection in a Maxwellian viscoelastic fluid through porous medium in the presence of magnetic field.

Formulation of the problem

Consider an infinite layer of an incompressible, thermally conducting Maxwellian viscoelastic fluid, confined between two horizontal planes situated at $z=0$ and $z=d$, acted upon by a uniform vertical magnetic field $\vec{H}(0,0,H)$, H is constant. The temperature T and solute concentrations $C^{(1)}$ and $C^{(2)}$ at the bottom and top surfaces $z=0$, $z=d$ are T_0 and T_1; $C_0^{(1)}$, $C_1^{(1)}$ and $C_0^{(2)}$, $C_1^{(2)}$ respectively, and a uniform temperature gradient $\beta\left(=\left|dT/dz\right|\right)$ and uniform solute gradients $\beta'\left(=\left|dC^{(1)}/dz\right|\right)$ and $\beta''\left(=\left|dC^{(2)}/dz\right|\right)$ are maintained. The gravity field $\vec{g}\left(0,0,-g\right)$ pervades the system. When the fluid flows through a porous medium, the gross effect is represented by Darcy's law, the equations of motion and continuity for Maxwellian viscoelastic fluid through porous medium following Boussinesq approximation are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial\vec{v}}{\partial t}+\frac{1}{\varepsilon}(\vec{v}.\nabla)\vec{v}\right]=-\frac{1}{\rho_0}\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla p+\mathbf{g}\left(1+\lambda\frac{\partial}{\partial t}\right)\left(1+\frac{\delta\rho}{\rho_0}\right)$$
$$+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}-\frac{\nu}{k_1}\vec{v}, \tag{1}$$

$$\nabla.\vec{v}=0, \tag{2}$$

where $\vec{v}$ is the filter velocity, ε is medium porosity, k_1 is the medium permeability and $\nu\left(=\mu/\rho\right)$. The fluid velocity $\vec{q}$ and the Darcian (filter) velocity $\vec{v}$ are connected by the relation $\vec{q}=\vec{v}/\varepsilon$. A porous medium of very low permeability allows us to use the Darcy's model. For a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy's model. This is because the viscous drag force is negligibly small in comparison with Darcy's resistance due to the large particle suspension.

When the fluid flows through a porous medium, the equation of heat conduction is

$$(\rho c_f\varepsilon+\rho_s c_s(1-\varepsilon))\frac{\partial T}{\partial t}+\rho c_f(\vec{v}.\nabla)T=\kappa\nabla^2 T \tag{3}$$

and analogous solute concentration equations are

$$(\rho c_f{}'\varepsilon+\rho_s c_s{}'(1-\varepsilon)\frac{\partial C^{(1)}}{\partial t}+\rho c_f{}'(\vec{v}.\nabla)C^{(1)}=\kappa'\nabla^2 C^{(1)} \tag{4}$$

$$(\rho c_f{}''\varepsilon+\rho_s c_s{}''(1-\varepsilon)\frac{\partial C^{(2)}}{\partial t}+\rho c_f{}''(\vec{v}.\nabla)C^{(2)}=\kappa''\nabla^2 C^{(2)}. \tag{5}$$

The Maxwell's equations yield

$$\varepsilon\frac{d\vec{H}}{dt} = (\vec{H}.\nabla)\vec{v}+\varepsilon\eta\nabla^2\vec{H}, \tag{6}$$

$$\nabla.\vec{H}=0. \tag{7}$$

Since density variations are mainly due to variations in temperature and solute concentrations, the equation of state for the fluid is given by

$$\rho=\rho_0[1-\alpha(T-T_a)+\alpha'(C-C_a^{(1)})+\alpha''(C-C_a^{(2)})], \tag{8}$$

where $\rho,\rho_0,t,\nu,\kappa,\kappa',\kappa'',\alpha,\alpha'$ and α'' are the fluid density, reference density, time, the kinematic viscosity, the thermal diffusivity, the solute diffusivities, thermal and solvent coefficients of expansion respectively. T_a is the average temperature given by $T_a=\left(T_0+T_1\right)/2$ where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer and $C_a^{(1)}$, $C_a^{(2)}$ are the average concentrations given by $C_a^{(1)}=\left(C_0^{(1)}+C_1^{(1)}\right)/2$ and $C_a^{(2)}=\left(C_0^{(2)}+C_1^{(2)}\right)/2$, where $C_0^{(1)}$, $C_1^{(1)}$ and $C_0^{(2)}, C_1^{(2)}$ are constant average concentrations of the lower and upper surfaces of the layer. Here $E=\varepsilon+(1-\varepsilon)\rho_s c_s/\rho c_f$ is a constant, E' and

E'' are analogous to E but corresponding to solute rather than heat. ρ, c_f ; ρ_s, c_s stand for density and heat capacity of fluid and solid matrix, respectively.

Basic state and perturbation equations

The basic state was assumed to be quiescent and is given by

$\vec{v} = (0, 0, 0),\ \vec{H}_b = (0, 0, H),\ T = T_b(z),\ p = p_b(z),\ C^{(1)} = C_b^{(1)}(z),\ C^{(2)} = C_b^{(2)}(z),$

$\rho = \rho_b(z), T_b(z) = T_a - \beta z, C_b^{(1)}(z) = C_a^{(1)} - \beta' z,\ C_b^{(2)}(z) = C_a^{(2)} - \beta'' z$ with

$$\rho_b = \rho_0 [1 - \alpha(T_b - T_a) + \alpha'(C_b^{(1)} - C_a^{(1)}) + \alpha''(C_b^{(2)} - C_a^{(2)})]. \tag{9}$$

Here we assume small perturbations on the basic state solution. Let $\vec{v}(u, v, w) = 0 + \vec{v}(u', v', w')$, $\rho = \rho_b + \rho'$, $\vec{H} = H_b + H'(h_x, h_y, h_z)$, $p = p_b + p'$, $T = T_b + T'$, $C^{(1)} = C_b^{(1)} + C^{(1)\prime}$ and $C^{(2)} = C_b^{(2)} + C^{(2)\prime}$ denote, respectively the perturbations in the fluid velocity, density, magnetic field, pressure, temperature and concentrations. The change in density ρ' caused mainly by the perturbations in temperature and concentrations is given by

$$\rho' = -\rho_0 [\alpha T' - \alpha' C^{(1)\prime} - \alpha'' C^{(2)\prime}]. \tag{10}$$

Since the non-linear theories attempt to allow for the finite amplitudes of the perturbations, we suppose that the various physical variables describing the flow suffer small (infinitesimal) increments and, as a consequence, we neglect all product and powers (higher than the first) of the increments and retained only terms that are linear and the linear stabilizing theory, for mathematical simplicity, is applied. Then the linearized hydromagnetic perturbation equations are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \vec{v}'}{\partial t} = -\frac{1}{\rho_0}\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla p' + g\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\alpha T' - \alpha' C^{(1)\prime} - \alpha'' C^{(2)\prime}\right) + \frac{\mu_e}{4\pi\rho_0}\left(\nabla \times H'\right)\times \vec{H} - \frac{\nu}{k_1}\vec{v}', \tag{11}$$

$$\nabla.\vec{v}' = 0, \tag{12}$$

$$E\frac{\partial T'}{\partial t} = \beta w + \kappa \nabla^2 T', \tag{13}$$

$$E'\frac{\partial C^{(1)\prime}}{\partial t} = \beta' w + \kappa' \nabla^2 C^{(1)\prime}, \tag{14}$$

$$E''\frac{\partial C^{(2)\prime}}{\partial t}=\beta'' w+\kappa''\nabla^2 C^{(2)\prime}, \tag{15}$$

$$\varepsilon\frac{dH'}{dt} = (H.\nabla)\vec{v}+\varepsilon\eta\nabla^2 H', \tag{16}$$

$$\nabla . H'=0. \tag{17}$$

Analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$\left[w,T',C^{(1)\prime},C^{(2)\prime},h_z\right]=\left[W(z),\Theta(z),\Gamma(z),\Psi(z),K(z)\right]\exp\{ik_x x+ik_y y+nt\}, \tag{18}$$

where k_x and k_y are the wave numbers in x and y directions respectively, $k=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. Using equation (18), equation (11) to (17) in non-dimensional form become

$$\left[\frac{\sigma}{\varepsilon}(1+\sigma F)+\frac{1}{p_l}\right](D^2-a^2)W+(1+\sigma F)\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma-\alpha''\Psi)$$

$$+(1+\sigma F)\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK=0, \tag{19}$$

$$\left(D^2-a^2-E\sigma p_1\right)\Theta=-\frac{\beta d^2}{\kappa}W, \tag{20}$$

$$\left(D^2-a^2-E'\sigma q_1\right)\Gamma=-\frac{\beta' d^2}{\kappa'}W, \tag{21}$$

$$\left(D^2-a^2-E''\sigma q_2\right)\Psi=-\frac{\beta'' d^2}{\kappa''}W, \tag{22}$$

$$\left(D^2-a^2-\sigma p_2\right)K=-\frac{Hd}{\varepsilon\eta}DZ. \tag{23}$$

Here we have put $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $F=\frac{\lambda\nu}{d^2}$, $p_1=\frac{\nu}{\kappa}$, $p_2=\frac{\nu}{\eta}$, $q_1=\frac{\nu}{\kappa'}$, $q_2=\frac{\nu}{\kappa''}$, $p_l=\frac{k_1}{d^2}$ and $D^*=dD$ [(*) is dropped for convenience]. Here we consider the case where both boundaries are free as well perfect conductors of both heat and solute concentrations, while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which equations (19) to (23) must be solved, are

$W=0,\ D^2W=0,\ \Theta=0,\ \Gamma=0,\ \Psi=0$ at $z=0$ and $z=1$.

$K=0$ on a perfectly conducting boundaryand h_x, h_y, h_z are continuous with an external vacuum field on a non-conducting boundary. (24)

Eliminating various physical parameters from equations (19) to (23), we obtain the final stability governing equation as

$$\left\{\frac{\sigma}{\varepsilon}(1+\sigma F)+\frac{1}{p_l}\right\}\left(D^2-a^2-E\sigma p_1\right)\left(D^2-a^2-E'\sigma q_1\right)\left(D^2-a^2-E''\sigma q_2\right)$$
$$\left(D^2-a^2-\sigma p_2\right)\left(D^2-a^2\right)W-Ra^2(1+\sigma F)\left(D^2-a^2-E'\sigma q_1\right)\left(D^2-a^2-E''\sigma q_2\right)$$
$$\left(D^2-a^2-E\sigma p_1\right)W+Sa^2(1+\sigma F)\left(D^2-a^2-E\sigma p_1\right)\left(D^2-a^2-E''\sigma q_2\right)\left(D^2-a^2-\sigma p_2\right)W$$
$$+S_1a^2(1+\sigma F)\left(D^2-a^2-E\sigma p_1\right)\left(D^2-a^2-E'\sigma q_1\right)\left(D^2-a^2-\sigma p_2\right)W$$
$$+\frac{Q(1+\sigma F)}{\varepsilon}\left(D^2-a^2-E\sigma p_1\right)\left(D^2-a^2-E'\sigma q_1\right)\left(D^2-a^2-E''\sigma q_2\right)\left(D^2-a^2\right)D^2W=0. \quad (25)$$

Here, $R=\frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $S=\frac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the analogous solute Rayleigh number, $S_1=\frac{g\alpha''\beta'' d^4}{\nu\kappa''}$ is another analogous solute Rayleigh number and $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is Chandrasekhar number.

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmosphere (Spiegel, [13]). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and $z=1$and hence the proper solution ofequation (25) characterizing the lowest mode is

$$W=W_0\sin\pi z, \quad (26)$$

where W_0 is constant. Substituting the proper solution (26) in equation (25), we obtain the dispersion relation

$$R_1=\left(\frac{1+\omega}{\omega}\right)\left[\left\{\frac{i\sigma_1}{\varepsilon}(1+i\sigma_1\pi^2F)+\frac{1}{P}(1+i\sigma_1\pi^2F_0)\right\}\frac{(1+\omega+i\sigma_1Ep_1)}{(1+i\sigma_1\pi^2F)}+S_2\left(\frac{1+\omega+i\sigma_1Ep_1}{1+\omega+i\sigma_1E'q_1}\right)\right.$$
$$\left.+S_3\left(\frac{1+\omega+i\sigma_1Ep_1}{1+\omega+i\sigma_1E''q_2}\right)+\frac{Q_1}{\varepsilon}\left(\frac{1+\omega}{\omega}\right)\left(\frac{1+\omega+i\sigma_1Ep_1}{1+\omega+i\sigma_1p_2}\right)\right]. \quad (27)$$

Here $R_1=\frac{R}{\pi^4}$, $S_2=\frac{S}{\pi^4}$, $S_3=\frac{S_1}{\pi^4}$, $Q_1=\frac{Q}{\pi^2}$, $\omega=\frac{a^2}{\pi^2}$, $i\sigma_1=\frac{\sigma}{\pi^2}$ and $P=\pi^2p_l$.

Stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Putting $\sigma=0$, the dispersion relation (27) reduces to

$$R_1=\frac{(1+\omega)}{\omega}\left(\frac{(1+\omega)}{P}+\frac{Q_1}{\varepsilon}\right)+S_2+S_3. \quad (28)$$

Thus, for the case of stationary convection, the relaxation time parameter F vanishes with σ and Maxwellian viscoelastic fluid behaves like an ordinary Newtonian fluid. The above relation expresses the modified Rayleigh number R_1 as a function of the parameters Q_1, S_2, S_3, P and dimensionless wave number ω. To study the effect of magnetic field, solute gradients and medium permeability, we examine the nature of $\frac{dR_1}{dQ_1}, \frac{dR_1}{dS_2}, \frac{dR_1}{dS_3}$ and $\frac{dR_1}{dP}$ analytically.

Equation (28) yields

$$\frac{dR_1}{dQ_1}=\frac{(1+\omega)}{\omega\varepsilon}, \quad (29)$$

which shows that magnetic field has stabilizing effect on the triple diffusive convection in Maxwellian viscoelastic fluid through porous medium.

From equation (28), we have

$$\frac{dR_1}{dS_2}=1 \text{ and } \frac{dR_1}{dS_3}=1, \quad (30)$$

which show that solute gradients have stabilizing effect on the triple diffusive convection.

It also follows from equation (23) that

$$\frac{dR_1}{dP} = -\frac{(1+\omega)^2}{\omega P^2}, \tag{31}$$

which is always negative. The medium permeability therefore, has destabilizing effect on the triple diffusive convection.

Some important theorems

Theorem 1: The system is stable or unstable.

Proof: Multiplying equation (19) by W^*, the complex conjugate of W, integrating over the range of z and making use of equations (20) to (23) together with the boundary conditions (24), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{p_l}\left(\frac{1}{1+\sigma F}\right)\right] I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}[I_2 + \sigma^* E p_1 I_3] + \frac{g\alpha'\kappa' a^2}{\nu\beta'}[I_4 + \sigma^* E' q_1 I_5]$$
$$+ \frac{g\alpha''\kappa'' a^2}{\nu\beta''}[I_6 + \sigma^* E'' q_2 I_7] + \frac{\mu_e \varepsilon\eta}{4\pi\rho_0\nu}[I_8 + \sigma^* p_2 I_9] = 0, \tag{32}$$

where $I_1 = \int\left(|DW|^2 + a^2|W|^2\right)dz$, $I_2 = \int\left(|D\Theta|^2 + a^2|\Theta|^2\right)dz$, $I_3 = \int |\Theta|^2\, dz$,

$I_4 = \int\left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz$, $I_5 = \int |\Gamma|^2\, dz$, $I_6 = \int\left(|D\psi|^2 + a^2|\psi|^2\right)dz$,

$I_7 = \int |\psi|^2\, dz$, $I_8 = \int\left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz$,

$I_9 = \int\left(|DK|^2 + a^2|K|^2\right)dz$,

and σ^* is the complex conjugate of σ. The integrals I_1 to I_9 are all positive. Putting $\sigma = \sigma_r + i\sigma_i$ in equation (32), where σ_r and σ_i are real and then equating the real and imaginary parts, we get

$$\left[\frac{\sigma_r}{\varepsilon} + \frac{1}{p_l}\left(\frac{1+\sigma_r F}{(1+\sigma_r F)^2 + \sigma_i^2 F^2}\right)\right] I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}\left(I_2 + \sigma_r E p_1 I_3\right)$$
$$+ \frac{g\alpha'\kappa' a^2}{\nu\beta'}[I_4 + \sigma_r E' q_1 I_5] + \frac{g\alpha''\kappa'' a^2}{\nu\beta''}[I_6 + \sigma_r E'' q_2 I_7] + \frac{\mu_e \varepsilon\eta}{4\pi\rho_0\nu}[I_8 + \sigma_r p_2 I_9] = 0. \tag{33}$$

and
$$\left[\left\{\frac{1}{\varepsilon}-\frac{1}{p_l}\left(\frac{F}{\left((1+\sigma_r F)^2+\sigma_i^2 F^2\right)}\right)\right\}I_1+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3-\frac{g\alpha'\kappa' a^2}{\nu\beta'}E'q_1I_5 -\frac{g\alpha''\kappa'' a^2}{\nu\beta''}E''q_2I_7-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2I_9\right]\sigma_i=0. \quad (34)$$

It is evident from equation (33) that σ_r may be negative or positive. The system is, therefore, stable or unstable.

Theorem 2: The modes may be oscillatory or non-oscillatory in contrast to case of no magnetic field and in the absence of solute gradients where modes are non-oscillatory.

Proof: Equation (34) yields that σ_i may be zero or non-zero, which means that the modes may be non-oscillatory or oscillatory. Further, in the absence of magnetic field and solute gradients, equation (34) reduces to

$$\left[\left\{\frac{1}{\varepsilon}-\frac{1}{p_l}\left(\frac{F}{\left((1+\sigma_r F)^2+\sigma_i^2 F^2\right)}\right)\right\}I_1+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3\right]\sigma_i=0. \quad (35)$$

For the condition $\frac{1}{\varepsilon}>\frac{1}{p_l}\left(\frac{F}{\left((1+\sigma_r F)^2+\sigma_i^2 F^2\right)}\right)$, the coefficient of σ_i in (35) is a positive definite and hence implies that $\sigma_i=0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. So, we can say that oscillatory modes are introduced due to the presence of magnetic field and stable solute gradients, which were non-existent in their absence.

Theorem 3: The sufficient conditions for the non-existence of overstability are

$\frac{1}{\varepsilon}>\frac{\nu\lambda}{k_1}$, $\frac{E}{\kappa}>\frac{1}{\eta}$, $\frac{E}{\kappa}>\frac{E'}{\kappa'}$ and $\frac{E}{\kappa}>\frac{E''}{\kappa''}$.

Proof: It is clear from equation (27) that R_1 will be complex for an assigned σ_1 while R_1 is real. Therefore, the condition that R_1 be real gives a relation between real and imaginary part of R_1. Assuming σ_1 is real and equating the real and imaginary parts of equation (27) and eliminating R_1 between them, we obtain

$$A_4c_1^4+A_3c_1^3+A_2c_1^2+A_1c_1+A_0=0, \tag{36}$$

where $c_1=\sigma_1^2$, $b=1+\omega$ and

$$A_4=\frac{F^2\pi^4E'^2E''^2q_1^2p_2^2q_2^2}{\varepsilon}b^2(b-1), \tag{37}$$

$$A_3=\Bigg[\frac{F^2\pi^4p_2^2(E'^2q_1^2+E''^2q_2^2)b^4}{\varepsilon}+\frac{F^2\pi^4E'^2E''^2q_1^2q_2^2b^4}{\varepsilon}$$

$$+\frac{F^2\pi^4QE'^2E''^2q_1^2q_2^2(Ep_1-p_2)b^2}{\varepsilon}+\frac{EF\pi^4E'^2E''^2F_0p_1q_1^2q_2^2b^3}{P}+\frac{EE'^2E''^2p_1p_2^2q_1^2q_2^2b}{P}$$

$$+E'^2E''^2p_2^2q_1^2q_2^2b^2\left(\frac{1}{\varepsilon}-\frac{\pi^2F}{P}\right)+\Big(F^2\pi^4E''^2p_2^2q_2^2S_2\left(Ep_1-E'q_1\right)+F^2\pi^4E'^2p_2^2q_1^2S_3$$

$$\left(Ep_1-E''q_2\right)\Big)b(b-1)\Bigg](b-1), \tag{38}$$

$$A_0=\Bigg[\left(\frac{1}{\varepsilon}+\frac{\pi^2F}{P}\right)b^8+\frac{Q(Ep_1-p_2)b^6}{\varepsilon}+\frac{Ep_1b^7}{P}+\Big(S_2(Ep_1-E'q_1)b^5$$

$$+S_3(Ep_1-E''q_2)b^5\Big)(b-1)\Bigg](b-1)=0. \tag{39}$$

The coefficients A_1 and A_2 being quite lengthy and not needed in the discussion of overstability, have not been written here.

Since σ_1 is real for overstability i.e. the four values of $c_1\,(=\sigma_1^2)$ should be positive. The sum of the roots of equation (36) which is $-\frac{A_3}{A_4}$, should be positive and product of the roots is $\frac{A_0}{A_4}$.

From the expressions (37) to (39), It is clear that A_4 is always positive and A_3 and A_0 are positive if

$\frac{1}{\varepsilon}>\frac{\pi^2F}{P}$, $Ep_1>p_2$, $Ep_1>E'q_1$ and $Ep_1>E''q_2$.

i.e. if $\frac{1}{\varepsilon}>\frac{\nu\lambda}{k_1}$, $\frac{E}{\kappa}>\frac{1}{\eta}$, $\frac{E}{\kappa}>\frac{E'}{\kappa'}$ and $\frac{E}{\kappa}>\frac{E''}{\kappa''}$. (40)

Thus, for the conditions (40), overstability cannot occur and the principle of exchange of stabilities is valid. Hence, these are the sufficient conditions for the non-existence of overstability.

Numerical results and discussion

For the stationary convection critical thermal Rayleigh number for the onset of instability is determined for critical wave number obtained by the condition $\frac{dR_1}{d\omega} = 0$ and analyzed numerically using Newton-Raphson method.

In Fig. 1, critical Rayleigh number R_1 is plotted against solute gradient parameter S_3 for fixed values of $S_2 = 40, Q_1 = 50, \varepsilon = 0.5$ and $P = 0.1, 0.5, 0.9$. The critical Rayleigh number R_1 increases with increase in solute gradient parameter S_3 which shows that solute gradient has stabilizing effect on the system.

In Fig. 2, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $S_2 = 20,\ S_3 = 20,\ \varepsilon = 0.5$ and $Q_1 = 100,\ 300,\ 500$. The critical Rayleigh number R_1 decreases with increase in medium permeability P which shows that medium permeability has destabilizing effect on the system.

In Fig. 3, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $S_2 = 20, S_3 = 20,\ \varepsilon = 0.5$ and $Q_1 = 10,\ 50,\ 90$. The critical Rayleigh number R_1 decreases with increase in medium permeability P which shows that medium permeability has destabilizing effect on the system.

In Fig. 4, critical Rayleigh number R_1 is plotted against magnetic field Q_1 for fixed value of $S_2 = 20, S_3 = 20,\ \varepsilon = 0.5$ and $P = 0.01,\ 0.05,\ 0.09$. The critical Rayleigh number R_1 increases with increase in magnetic field Q_1 which shows that magnetic field has stabilizing effect on the system.

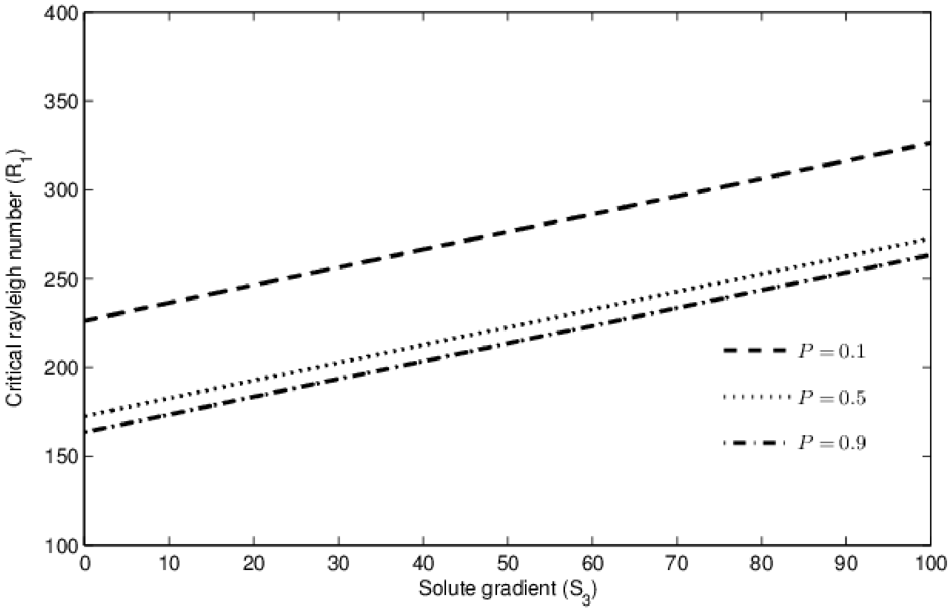

Fig. 1: Variations of critical Rayleigh number R_1 with S_3 for fixed value of $S_2 = 40$ $Q_1 = 50$, $\varepsilon = 0.5$ and $P = 0.1, 0.5, 0.9$.

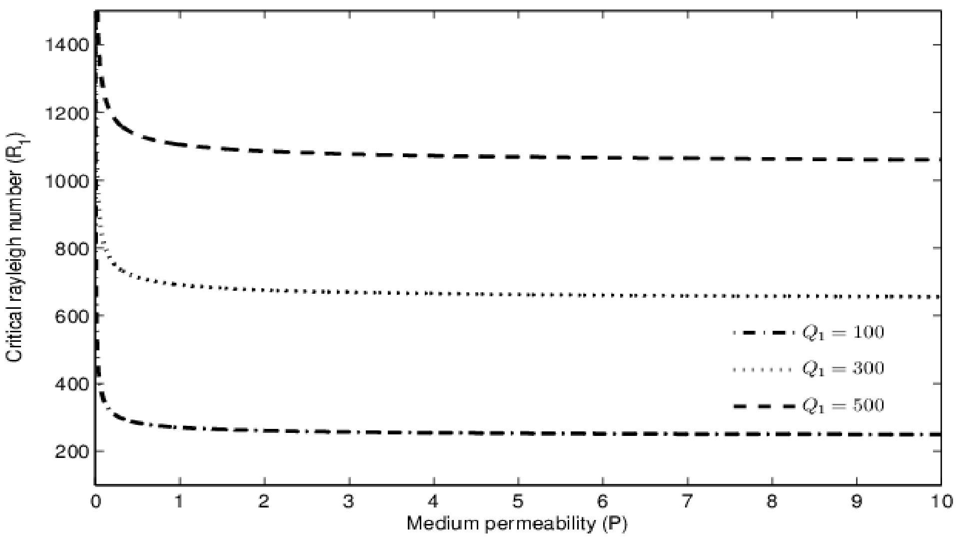

Fig. 2: Variations of critical Rayleigh number R_1 with P for fixed value of $S_2 = 20, S_3 = 20$, $\varepsilon = 0.5$ and $Q_1 = 100$, 300, 500.

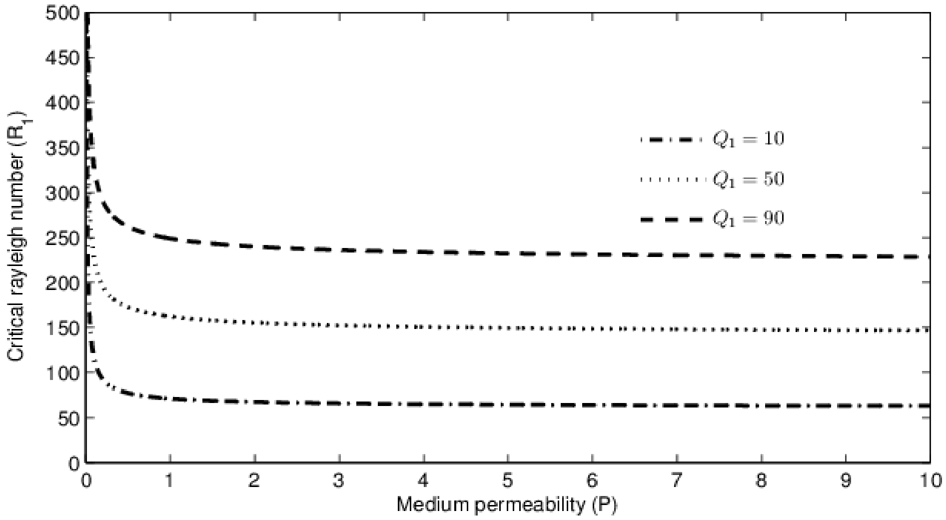

Fig. 3: Variations of critical Rayleigh number R_1 with P for fixed value of $S_2 = 20$, $S_3 = 20$, $\varepsilon = 0.5$ and $Q_1 = 10, 50, 90$.

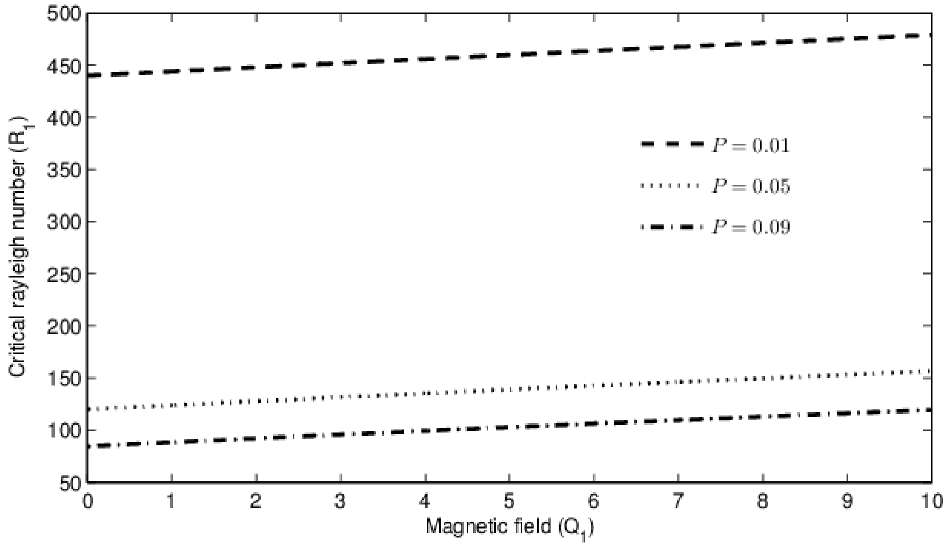

Fig. 4: Variations of critical Rayleigh number R_1 with Q_1 for fixed value of $S_2 = 20$, $S_3 = 20$, $\varepsilon = 0.5$ and $P = 0.01, 0.05, 0.09$.

Conclusions

The subject of double-diffusive convection is still an active research area, however, there are many fluid dynamical systems occurring in nature and industrial applications involve three or more stratifying agencies having different molecular diffusivities. More complicated systems can be found in magmas and molten metals. This has prompted researchers to study convective instability in triply diffusive fluid systems. Motivated by this, the effect of uniformvertical magnetic field on triply diffusive convection in a layer of Maxwellian viscoelastic fluid heated and soluted from below is considered in the present paper. The main conclusions from the analysis of this paper are as follows:

(a) For the case of stationary convection the following observations are made:

- The relaxation time parameter F vanishes with σ and so Maxwellian viscoelastic fluid behaves like an ordinary Newtonian fluid.
- The magnetic field and solute gradients have stabilizing effect, whereas the medium permeability has destabilizing effect on the system.

(b) It is observed that solute gradients and magnetic field introduce oscillatory modes in the system, which was non-existent in their absence.

(c) The sufficient conditions for the non-existence of overstability are

$$\frac{1}{\varepsilon} > \frac{\nu\lambda}{k_1}, \ \frac{E}{\kappa} > \frac{1}{\eta}, \frac{E}{\kappa} > \frac{E'}{\kappa'} \text{ and } \frac{E}{\kappa} > \frac{E''}{\kappa''}.$$

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