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Optimal Inventory Policies for Single-Supplier Single-Buyer Deteriorating Items with Price-Sensitive Stock-dependent Demand and Order Linked Trade Credit

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Abstract

The integrated inventory policies of single-supplier single-buyer are studied under order-dependent trade credit terms. The units in inventory are subject to constant rate of deterioration. The demand rate is assumed to be price-sensitive stock-dependent. Under order-dependent trade credit scenario, the buyer is eligible for settlement of account at allowable delay period if the order quantity is larger than that of pre-specified order by the supplier. The objective is to maximize joint profit of the supply chain with respect to retail price of the unit and cycle time and number of shipments from the supplier to the buyer. The numerical example is given to illustrate the mathematical analysis. Sensitivity analysis is carried out to deduce managerial insights

Keywords: Integrated inventory model, price-sensitive stock-dependent demand, deterioration, order-dependent trade credit

1. Introduction

The classical economic order quantity (EOQ) model considered that the demand rate of an item is constant in the period under review and the buyer will pay for items immediately. Both these assumption are relaxed by number of researchers to discuss variants of EOQ model. Goyal (1985) incorporated the concept of delay payment in EOQ model. This model was formulated under the assumption of constant demand. When buyer is offered a credit period to settle the account, he can earn interest on the sales revenue by selling the items. Goyal (1985) calculated interest earned on unit purchase price. Thereafter, researchers analyzed this promotional tool to study the advantage for the buyer. The literature survey on inventory models and trade credit by Shah *et al.* (2010) gives the modelling and analysis of this promotional tool widely used by the supplier to stimulate the demand, to lower the investment and attract the more buyers. In the cited articles, the offer of trade credit from the supplier to the buyer does not consider the size of the order.

Khouja and Mehraj (1996) discussed vendor's credit polices to compute order quantity when credit terms are order dependent. Shinn and Hwang (2003), Chang (2004), Chang *et al.* (2005) Chang and Liao (2004, 2006), Chang *et al.* (2009), Shah and Shukla (2010, 2011), Shah (2010), Shah *et al.*(2012) incorporated order-linked trade credit and different demand structure in their analysis. In these models, a single player was a decision maker.

Globalization of the business brought the players to work jointly. Goyal (1976) developed a single-vendor single-buyer integrated inventory model. Banerjee (1986) analyzed integrated decision when vendor follows lot-for-lot production policy. Goyal (1988) relaxed the assumption of lot-for-lot production policy and established that vendor's economic production quantity should be an integral multiple of buyer's order size. Lu (1995), Goyal (1995), Vishwanathan (1998), Hill (1997,1999), Yang and Wee (2003) encouraged more batching and frequent ordering for the players of the supply chain.

In this research, Levin *et al.* (1972)'s quote "large piles of goods attract more customers" is incorporated in the form of stock-dependent demand. The integrated inventory system of single-vendor single-buyer for single-item which deteriorate at a constant rate is considered. The production is proportional to demand to avoid shortages. The buyer is eligible for the offer of trade credit if the order size is greater than or equal to that of pre-specified order size by the vendor. The total joint profit per unit time of the supply chain is maximized with respect to retail price of the unit, order quantity for the buyer and number of transfers from the vendor to the buyer. An algorithm is proposed to determine the best decision policy. The numerical example is given to validate the mathematical development of the problem. The managerial observations are derived sensitivity analysis.

2. Notations and Assumptions

a) Notations

$R(I(t), s)$ Price-sensitive Stock-dependent demand rate; $(\alpha+\beta I(t))s^{-\eta}$ where $\alpha>0$ denotes constant scale demand, $0<\beta<1$ denotes stock-dependent parameter, $\eta>1$ denotes price-elasticity and s denotes retail price of the product per unit by the buyer. (s is a decision variable)

θ Deterioration rate of units in inventory system; $0<\theta<1$

A_v Vendor's set-up cost per set-up

A_b Buyer's ordering cost per order

C_p Production cost per unit

C_b Buyer's purchase cost per unit

Note: $s > C_b > C_p$

I_v Vendor's inventory holding cost rate per unit per year, excluding interest charges

I_b Buyer's inventory holding cost rate per unit per year, excluding interest charges

I_{vp} Vendor's opportunity cost /$/ unit time

I_{bp} Buyer's opportunity cost /$/ unit time

I_{be} Buyer's interest earned /$/ unit time

ρ Capacity utilization which is ratio of demand rate to the production rate; $\rho<1$ is known constant

M Permissible credit period for the buyer given by the vendor

Q Buyer's order quantity per order (a decision variable)

Q_d Pre-specified order quantity to qualify for delayed payment

T Cycle time (a decision variable)

T_d The duration of time when Q_d - units are sold off

n Number of transfers from the vendor to the buyer (a decision variable)

TVP Vendor's total profit per unit time

TBP Buyer's total profit per unit time

π $(=TBP+TVP)$ Joint total profit per unit time

b) Assumptions

The following assumptions are made in deriving the proposed model.

1. Single - vendor and single - buyer supply chain is studied for a single-item.
2. Lead - time is zero. Shortages are not allowed.
3. The demand rate is price - sensitive stock-dependent.
4. The buyer qualifies for settlement of account at a later date if the order is equal or larger than the pre-specified quantity Q_d by the vendor. Otherwise, the buyer must pay for the purchases immediately.
5. During the credit period, the buyer earns interest at the rate I_{be} per unit on the generated revenue. At the end of the credit period, the buyer pays incurs an opportunity cost at a rate of I_{bp} on the unsold items in the warehouse.
6. The units in inventory are subject to deterioration at a constant rate 'θ', $0<\theta<1$. The deteriorated units can neither be repaired nor replaced during the cycle time.

3. Mathematical Model

In this section, we develop the integrated inventory policies when demand is price-sensitive stock-dependent demand and trade credit is offered only if buyer's order quantity is equal or greater than a pre-specified quantity.

a) Vendor's total profit per unit time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows: Sales revenue: The total sales revenue per unit time is

$$(C_b - C_p)\frac{Q}{T}. \text{ (See Appendix A for computation of } Q\text{)}$$

(1) Set-up cost: nQ-units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is $\frac{A_v}{nT}$.

(2) Holding cost: Using Joglekar (1988), the vendor's average inventory per unit time is $\frac{C_p(I_v + I_{vp})[(n-1)(1-\rho)+\rho)]}{T(\beta^2 + 2\beta\theta s^\eta + \theta^2 s^{2\eta}}\alpha\left(s^\eta\left(e^{(\beta s^{-\eta}+\theta)T} - 1 - T\theta\right) - \beta T\right)$

(3) Opportunity cost: If Q_d or more units are ordered by the buyer, the credit period of *M*–units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when $T \geq T_d$, the delay in payments is permissible and corresponding opportunity cost per unit time is $\frac{C_b I_{vp} QM}{T}$. On the other hand, when $T \geq T_d$ the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

$$TVP(n) = \begin{cases} TVP_1(n), \ T < T_d \\ TVP_2, \ T \geq T_d \end{cases} \quad \ldots(1)$$

where

$$TVP_1(n) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{vp})[(n-1)(1-\rho)+\rho]}{T\left(\beta^2 + 2\beta\theta s^\eta + \theta^2 s^{2\eta}\right)}\alpha\left(s^\eta\left(e^{\left(\beta s^{-\eta}+\theta\right)T} - 1 - T\theta\right) - \beta T\right) \quad \ldots(2)$$

$$TVP_2(n) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_b I_{vp} QM}{T} - \frac{C_p(I_v + I_{vp})[(n-1)(1-\rho)+\rho]}{T\left(\beta^2 + 2\beta\theta s^\eta + \theta^2 s^{2\eta}\right)}\alpha\left(s^\eta\left(e^{\left(\beta s^{-\eta}+\theta\right)T} - 1 - T\theta\right) - \beta T\right) \quad \ldots(3)$$

b) Buyer's total profit per unit time

The total profit per unit time for the buyer is sales revenue minus ordering cost, holding cost, and opportunity cost plus interest earned. These costs are computed as follows:

(1) Sales revenue: The total sales revenue per unit time is $(s-C_b)\frac{Q}{T}$. (See Appendix A for computation of *Q*)

(2) Ordering cost: The ordering cost per unit time is $\frac{A_b}{T}$

(3) Holding cost: The buyer's holding cost (excluding interest charges) per unit time is $\frac{C_b I_b}{T\left(\beta^2+2\beta\theta s^\eta+\theta^2 s^{2\eta}\right)}\alpha\left(s^\eta\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right)$

(4) Opportunity cost: Based on the lengths of *T, M* and T_d, the following four cases arises (i) $0<T<T_d$ (ii) $T_d \le T \le M$ (iii) $T_d \le M \le T$ (Fig. 1) and (iv) $M \le T_d \le T$

The cases (iii) and (iv) are similar. We discuss opportunity cost and interest earned in each case.

Opportunity cost per unit time

$$
=\begin{cases}
\frac{C_b I_{bp} Q}{T} & ,0<T<T_d \\
0 & ,T_d \le T \le M \\
\frac{C_b I_{bp}}{T}\left(\int_M^T I(t)\,dt\right) & ,T_d \le M \le T \text{ or } M \le T_d \le T
\end{cases}
$$

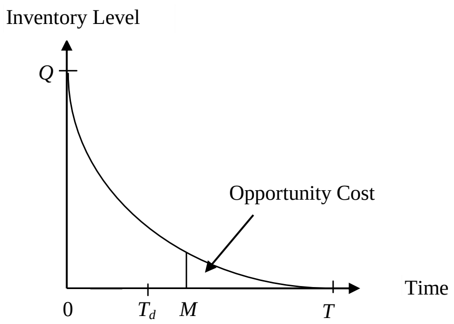

Figure.1 Opportunity cost for $T_d \le M \le T$ or $M \le T_d \le T$

Interest earned: As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time

$$=\begin{cases} 0 & , 0<T<T_d \text{ (because payment is to be made on delivery)} \\ \dfrac{sI_{be}}{T}\left(\int_0^T R(I(t),s)\,t\,dt+Q(M-T)\right) & , T_d \le T \le M \text{ (figure 2)} \\ \dfrac{sI_{be}}{T}\left(\int_0^M R(I(t),s)\,t\,dt\right) & , T_d \le M \le T \text{ or } M \le T_d \le T \text{ (figure 3)} \end{cases}$$

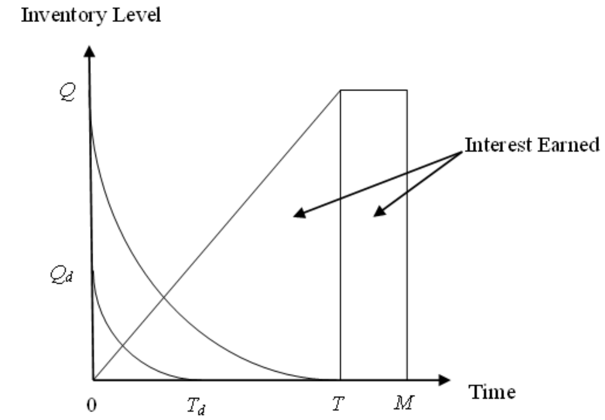

Figure.2 Interest earned by buyer when $T_d \le T \le M$

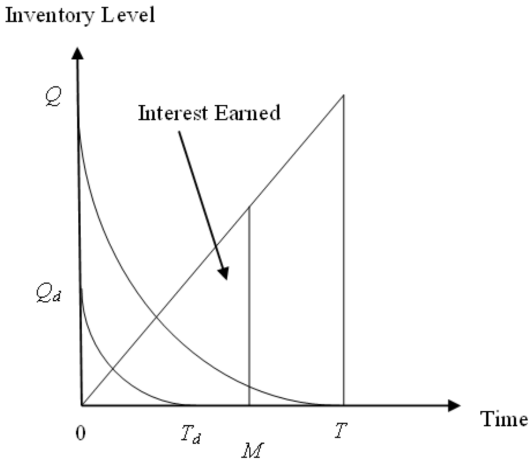

Figure.3 Interest earned by buyer when $T_d \le M \le T$

Hence, the buyer's total profit per unit time is

$$TBP(T)=\begin{cases} TBP_1(T),\ 0<T<T_d \\ TBP_2(T), T_d \le T \le M \\ TBP_3(T), T_d \le M \le T \\ TBP_4(T), M \le T_d \le T \end{cases} \quad \ldots(4)$$

where

$$TBP_1(T)=\frac{(s-C_b)Q}{T}-\frac{A_b}{T}-\frac{C_b I_b}{T\left(\beta^2+2\beta\theta s^{\eta}+\theta^2 s^{2\eta}\right)}\alpha\left(s^{\eta}\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right) -\frac{C_b I_{bp} Q}{T} \quad \ldots(5)$$

$$TBP_2(T)=\frac{(s-C_b)Q}{T}-\frac{A_b}{T}-\frac{C_b I_b}{T\left(\beta^2+2\beta\theta s^{\eta}+\theta^2 s^{2\eta}\right)}\alpha\left(s^{\eta}\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right) +\frac{sI_{be}}{T}\left(\int_0^T R\left(I(t),s\right)t\,dt+Q(M-T)\right) \quad \ldots(6)$$

$$TBP_3(T)=TBP_4(T)=\frac{(s-C_b)Q}{T}-\frac{A_b}{T}-\frac{C_b I_b}{T\left(\beta^2+2\beta\theta s^{\eta}+\theta^2 s^{2\eta}\right)}\alpha\left(s^{\eta}\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right) -\frac{C_b I_{bp}}{T}\left(\int_M^T I(t)dt\right)-\frac{sI_{be}}{T}\left(\int_0^M R\left(I(t),s\right)t\,dt\right) \quad \ldots(7)$$

C) Joint total profit per unit time

In the integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply chain with respect to retail price, cycle time and number of shipments from the vendor to the buyer. The joint total profit per unit time for the integrated system is

$$\pi(n,T)=\begin{cases} \pi_1(n,T)=TVP_1(n)+TBP_1(T), \quad 0<T<T_d \\ \pi_2(n,T)=TVP_2(n)+TBP_2(T),\ T_d \le T \le M \\ \pi_3(n,T)=TVP_2(n)+TBP_3(T),\ T_d \le M \le T \\ \pi_4(n,T)=TVP_2(n)+TBP_3(T),\ M \le T_d \le T. \end{cases} \quad \ldots(8)$$

where

$$\pi_1(n,T)=\left(s-C_p-C_bI_{bp}\right)\frac{Q}{T}-\frac{\bar{A}}{T}-\frac{1}{T}(\phi+\psi)\alpha\left(s^{\eta}\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right) \quad \ldots(9)$$

$$\pi_2(n,T)=\left(s-C_p-\left(C_bI_{vp}-sI_{be}\right)M\right)\frac{Q}{T}-sI_{be}Q-\frac{\bar{A}}{T}$$
$$-\frac{1}{T}(\phi+\psi)\alpha\left(s^{\eta}\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right)+\frac{sI_{be}}{T}\int_0^T R\left(I(t),s\right)t\,dt \quad \ldots(10)$$

$$\pi_3(n,T)=\left(s-C_p-C_bI_{vp}M\right)\frac{Q}{T}-\frac{\bar{A}}{T}-\frac{1}{T}(\phi+\psi)\alpha\left(s^{\eta}\left(e^{\left(\beta s^{-\eta}+\theta\right)T}-1-T\theta\right)-\beta T\right)$$
$$-\frac{C_b\,I_{bp}}{T}\left(\int_M^T I(t)\,dt\right)-\frac{sI_{be}}{T}\left(\int_0^M R\left(I(t),s\right)t\,dt\right) \quad \ldots(11)$$

Where

$$\bar{A}=A_b+\frac{A_v}{n}$$
$$\phi=\frac{C_p(I_v+I_{vp})\left[(n-1)(1-\rho)+\rho\right]}{\left(\beta^2+2\beta\theta s^{\eta}+\theta^2s^{2\eta}\right)}$$
$$\psi=\frac{C_b\,I_b}{\left(\beta^2+2\beta\theta s^{\eta}+\theta^2s^{2\eta}\right)}$$

4. Computational Procedure

For fixed *T*, we note that $\pi(n,T)$ is a concave function of *n* because $\frac{\partial^2\pi(n,T)}{\partial n^2}=-\frac{2A_v}{n^3T}<0$. Therefore to find optimum number of shipments n^* we will have a local optimal solution. The optimum value of cycle time and retail price can be obtained by setting $\frac{\partial\pi}{\partial T}=0$ *and* $\frac{\partial\pi}{\partial s}=0$ simultaneously for fixed *n*.

Algorithm:

Step 1: Set parametric values.

Step 2: Compute T_d using $\frac{1}{(\beta s^{-\eta}+\theta)} \ln\left(1+\frac{(\beta s^{-\eta}+\theta) Q_d}{\alpha s^{-\eta}}\right)$ for given value of Q_d.

Step 3: Set n =1.

Step 4: Knowing T_d and M, compute T and s by solving $\frac{\partial \pi_j}{\partial T}=0$ and $\frac{\partial \pi_j}{\partial s}=0$ simultaneously for $j=1, 2, 3$.

Step 5: Find corresponding profit π_j for $j=1, 2, 3$.

Step 6: Increment n by 1.

Step 7: Repeat step 4 - 6 until

$$\pi(n-1, T(n-1), s(n-1)) \leq \pi(n, T(n), s(n)) \geq (n+1, T(n+1), s(n+1))$$

Once the optimal solution (n^*, T^*, s^*) is obtained, the optimal order quantity can be obtained.

5. Numerical Examples and Interpretations

Example 1 Consider, α=10000 units, $\beta = 10\% = 0.1$, $\eta = 1.25$ $\theta = 0.1$, $\rho =$ 0.7, C_b = \$10/unit, C_p = \$5/unit, C_v = \$400/setup, C_b = \$50/order, I_v = 10%/unit/annum, I_b = 10%/unit/annum, $I_{bp} = 0.18$/ annum, $I_{be} = 0.12$/ annum, $I_{ve} = 0.05$, s = \$25/unit and M=30 days. The optimal shipments and ordering units with buyer, vendor and joint profit for different values of Q_d are exhibited in Table 1.

Table: 1 Optimal solutions for different Q_d

Q_d	Q^*	n^*	T^* **(days)**	**Profit(\$)**		
				Buyer	**Vendor**	**Joint**
100	54.93	13	135	2619	550	3169
200	54.93	13	135	2619	550	3169
300	300/66.71	11	168	2675	531	3206
400	400/66.71	11	168	2675	531	3206
500	54.93	13	135	2619	550	3169
600	54.93	13	135	2619	550	3169

From Table 1, it is seen that the vendor's profit and joint total profit of the system increase with increase in Q_d and then further increase in pre-specified units lower their profits whereas for the buyer, it decreases. It is seen that the buyer's optimal order quantity Q^* is equal to Q_d and less than Q_d when $Q_d \geq 500$ Thus, vendor is directed to set threshold which is favorable to both the players of the supply chain. If the threshold set by the vendor is too high, the buyer will falter to order a quantity greater than the threshold to take advantage of delayed payments. The concavity of joint total profit for (n, s) for obtained T is exhibited in fig. 4, for (n, T) for $s = 30.14$ in fig. 5 and for (s, T) for 11 - shipments in fig. 6.

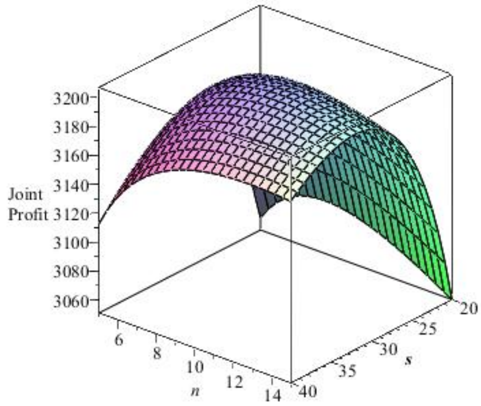

Figure 4 Concavity of joint total profit for (*n*, *s*) for obtained *T*

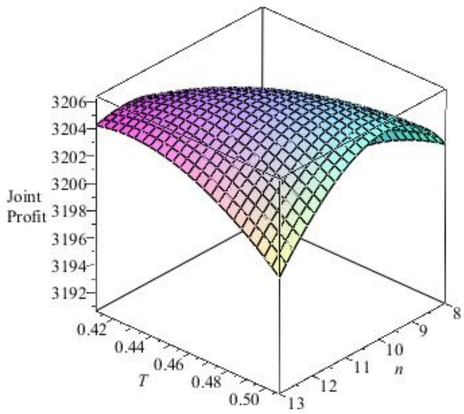

Figure 5 Concavity of joint total profit for (*n*, *T*) for obtained *s*

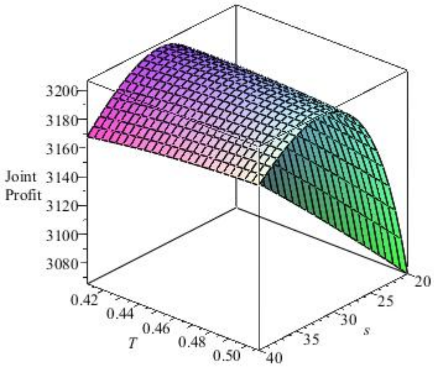

Figure 6 Concavity of joint total profit for (*s*, *T*) for obtained *n*

Example 2 Consider the data given in Example 1. We study the effect of delayed payments for $Q_d = 300$ units.

Table 2 Optimal solutions for different *M* ($Q_d = 300$)

M **(days)**	Q^*	n^*	T^* **(days)**	**Profit($)**		
				Buyer	**Vendor**	**Joint**
20	300	11	169	2668	532	3200
30	300	11	168	2675	531	3206
40	300	11	167	2682	531	3213
50	300	11	166	2689	531	3220
60	300	11	165	2696	531	3227

From Table 2, it is observed that buyer's total profit and joint profit of the supply chain increases with increase in the offered credit period. The longer credit period shrinks vendor's total profit because payment will be received late for the purchases made. This suggests that late payment increases risk of cash shortage for the vendor.

Example 3. In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in the optimal cycle time, purchase quantity and joint profit are studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and 20%. The results are exhibited in Figure 7.

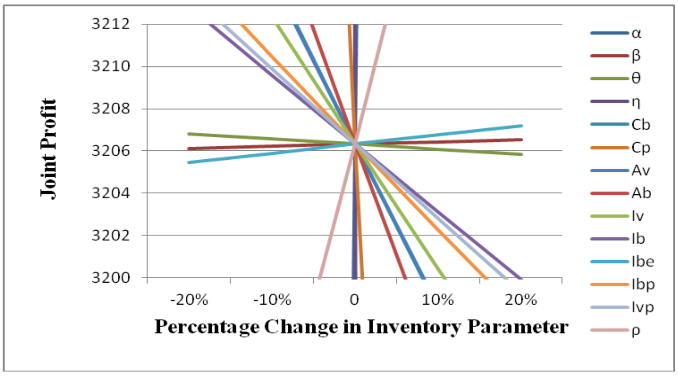

Fig. 7 Variations in joint total profit

It is observed from fig. 7 that the scale demand has positive impact on joint profit. It reveals that both the players should take advantage of demand increase and setting agreeable selling price. Production cost of supplier decreases joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small perturbations in profit of the supply chain. The deterioration of units in inventory also decrease the joint total profit of the supply chain.

Example 4. In table 3, we compare independent *v.s.* joint decision, for pre-specified quantity $Q_d = 300$ units at which buyer qualifies for delayed payment period. It is observed that the buyer's profit decreases in integrated decision while that of vendor increases significantly. This will discourage the buyer to opt for joint decision. To entice buyer for joint decision we give readjustments of profits (discussed in Goyal (1976)) in the last row of the table.

Table 3 Optimal Solution of independent and scenario

Scenario			Buyer	Vendor	Joint
Independent	Total Shipments	10			
	Ordering Quantity(units)	50			
	Cycle Time(in days)	258			
	Total Annual Profit(in $)		2879	218	3097
Integrated	Total Shipments	11			
	Ordering Quantity(units)	67			
	Cycle Time(in days)	168			
	Total Annual Profit(in $)		2675	531	3206
	Readjusted Total Annual Profit		2980	226	3206

where

Buyer's profit

$$=\pi(n,T)\times\frac{TBP(P,T)}{\left[TBP(P,T)+TVP(n)\right]}=3206\times\frac{2879}{(2879+218)}=2980$$

Supplier's profit

$$=\pi(n,T)\times\frac{TVP(n)}{\left[TBP(P,T)+TVP(n)\right]}=3206\times\frac{218}{(2879+218)}=226$$

Table 3 shows that the total annual profit under joint decision $3206 (=$2675+$531) which is greater than the total profit under independent decision $3097 (=$2879+$218). It establishes that joint decision is advantageous to both the players.

6. Conclusion

A single-vendor single-buyer inventory policy is analyzed. The demand is considered to be price-sensitive stock-dependent. The units in inventory deteriorate at a constant rate. The vendor offers order-linked credit time to settle the account for the purchases made. The computational algorithm is outlined to maximize the joint total profit per unit time with respect to number of transfers from the vendor to the buyer, retail price of the product to be set by the buyer and cycle time.

Based on the results, it is established that longer credit period helps buyer while out-flow risk is for vendor. To invite the buyer for the co-ordinate decision, vendor should set proper threshold for pre-specified order units. It is observed that the order - linked trade credit attracts the buyer for larger order and thereby saving in transportation cost which is part of the ordering cost.

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Appendix A

The rate of change of inventory at any instant of time can be discussed by differential equation

$$\frac{dI(t)}{dt} = -\Big(\big(\alpha + \beta I(t)\big)s^{-\eta} + \theta I(t)\Big), 0 \le t \le T$$

with $I(0) = Q$ and $I(T) = 0$. Using $I(T) = 0$, the solution of the differential equation is

$$I(t) = \frac{\alpha s^{-\eta}}{\beta s^{-\eta} + \theta}\left(e^{\left(\left(\beta s^{-\eta} + \theta\right)(T-t)\right)} - 1\right),\ 0 \le t \le T$$

The units to be purchased is given by

$$Q = I(0) = \frac{\alpha s^{-\eta}}{\beta s^{-\eta} + \theta}\left(e^{\left(\left(\beta s^{-\eta} + \theta\right)T\right)} - 1\right).$$