International Journal of Inventory Control and Management
ISSN-(Printing) 0974-7273, (Online) 0975-3761

EOQ Model with Natural Idle Time and Wrongly Measured Demand Rate

Sujit Kumar De*
Department of Mathematics, Midnapore College,
Pin-721101,West Bengal, India
E-mail:-skdemamo2008.com@gmail.com

IJICM

Volume 3 | Number 1-2 | Jan.-Dec. 2013
pp. 329-351

ARTICLE INFO
Article History
Received : 3-5-13
Accepted :17-8- 13

* Corresponding Author

Abstract

The traditional concept of economic order quantity (EOQ) inventory model is that, the demand rate is continuous throughout the cycle time. But through clinical inquiry anybody can notice something different from that assumption. In reality, for any inventory we usually observe that each day has an unavoidable natural leisure/closing time and at that time no demand rate is viewed. Considering this phenomenon in an EOQ model we may split a day into two parts, one is the opening time period and the other one is closing time period. However, the demand rate which is out-flowed from the system has a measuring error. In the basis of low, zero and excessive measuring error, the model itself is divided into three parts, for a finite cycle time (counted in days only). Due to the flexible nature of the opening and closing time period of the model, a general Fuzzy set and Intuitionistic Fuzzy Optimization (IFO) technique have been analyzed to illustrate the model. Numerical examples and sensitivity analysis have been made with the help of LINGO software and got a wonderful optimum solution near 12-hour opening time duration in a day. Finally a conclusion is made keeping some scope of future work.

Keywords: *Inventory, Fuzzy sets, Natural idle time, Intuitionistic Fuzzy Optimization.*

AACS

1. Introduction

In the classical inventory EOQ model, it was tacitly assumed that the system starts at time zero and continues up to cycle time without getting any pause in the intermediate time. Over the last four decades, a considerable number of research works has been done along this direction. The basic assumptions over the traditional EOQ model are demand rate is fixed and inventory run time (cycle time) is continuous in nature. But, in practice, in most of the cases the inventory run time throughout the cycle time is not continuous due to the existence of natural leisure/pause/idle time (for instance, 8:00 PM to 8:00AM in a day). However, in retail market, the essential commodities like vegetable, potato, onions, fruits etc. are not sold by sectioning the goods to meet the customers' specified amount on demand. So it is quite impossible to every seller of a market to measure the required items correctly. The customers are usually unwilling to pay more money for such excess amount as demanded by the seller. In many cases, for less measure demand quantity no bargaining is allowed with the seller and hence the customers are forced to pay the specified amount of money for their desired amount of demand. To overcome the customers' dissatisfactions, the sellers usually overlook the fact with tactful handling of items on demand or other several means (using defective weighing machine etc.) so that the customers are convinced. For example, at the most popular tourist spot like Kashmir (India) purchasing 20 Kilogram Apple, one gets only 15-18 Kilogram at home! Thus a motivation would come over the fact for which we may develop a model with wrongly measured demand rate. The errors come at the time of measuring the goods which may be negative or positive. Positive measuring error is usually called deterioration at the time of supplying demand. The conventional deteriorating inventory deals with the deterioration over inventory level. In the early stage, decaying inventory was developed by Ghare and Schrader (1963) and later Covert and Philip (1973) analyzed their model considering deterioration as two parameter Weibull distribution. Misra(1975) studied an economic production quantity(EPQ) model with variable rate of deterioration and finite rate of production. Periodic demand rate in EPQ system was considered by Banerjee(1992) and inventory level dependent demand rate with relaxed terminal conditions was developed by Urban(1972). A vendor-buyer co-operative model with stochastic demand was analyzed by Liang et.al.(2004).

Zadeh (1965) first developed the concept of fuzzy set theory and later on Bellmann and Zadeh (1970) made an application of fuzzy set theory in several decision making problems of Operations Research. Since then, numerous research findings have been established along this direction. Kaufmann and Gupta (1988) developed a fuzzy mathematical model in engineering and management science. De and Goswami assumed deterioration rate as fuzzy number in EPQ system. Kao and Hsu (2002) discussed a single period inventory model with fuzzy demand. Model with variable lead time and fuzzy lost sales are analyzed by Ouyang and Chang (2002). Probabilistic inventory model with fuzzy cost components was developed by Banerjee and Roy (2010 a) considering the random variable as fuzzy number.

In crisp sense, several optimization techniques have been used in the modern developmental inventory model. Some of them are, Golden Region Search method in Simulation technique that was developed by Kabirian and Olafsson (2011) and another one is an analytic approach via Eigen values of the system Jacobian Matrix expressing from Characteristic polynomial that was analyzed by Saleh et.al.(2010). But in the fuzzy environment, the early stage of Intuitionistic Fuzzy Set (IFS) developed by Atanassov (1986). The concepts of IFS can be viewed as an alternative approach to define a fuzzy set in the case where the information available in the system is not sufficient for the definition of an imprecise concept or ambiguity (vagueness) concerning the description of the semantic meaning of declaration of statements relating to an economic world in the sets itself. Therefore, it is to expect that, IFS can be used to simulate human decision making process and activities that requires human expertise and knowledge which is valid and reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity. Angelov (1997) implemented the IFS in optimizing the real world problem in Intuitionistic fuzzy environment. An interval valued Intuitionistic Fuzzy Sets was developed by Atanassov and Gargov (1989) and a solution of a probabilistic fixed order interval system was analyzed by Banerjee and Roy (2010 b) alone.

In this paper, an EOQ model has been developed under the natural leisure/pause/idle time, incorrect demand rate and without shortage. Assuming the inventory opening time period and closing time period per day as fuzzy numbers, a general fuzzy non-linear programming (GFNLP) and IFS approach have been analyzed. Numerical illustrations have been done by the use of algorithms and LINGO software. A sensitivity analysis and graphical trend of the model have been presented; finally, a conclusion is made putting some scope of future work.

2. Assumptions and Notations

The following notations and assumptions are used to develop the model.

Assumptions

1. The inventory starts at the beginning of the opening time and maintaining the natural leisure time it ends at beginning of the closing time of the last day of the cycle time.

2. Replenishments are instantaneous.
3. The time horizon is infinite (days)
4. The sum of opening and closing time period is unity.
5. Shortages are not allowed.
6. Demand rate per unit time is constant with error fraction θ.
7. Holding cost is uniform over the cycle time.
8. Average natural idle time (leisure/pause) cost is constant per unit idle time.
9. Order quantity received is exactly q (without measuring error).
10. Affective loss or profit is calculated for the case of incorrect demand rate only.
11. The security charge, telephone charge, transportation cost (if any) etc. beyond the working hours may be considered as the idle time cost.

Notations

i) q : The order quantity per cycle

ii) t_1: Duration of opening time (day)

iii) t_2 : Duration of closing /natural idle time (day)

iv) d : Demand rate per unit time in (0 , t_1)

v) θ: Error fraction in demand rate

vi) h : Inventory holding cost per unit quantity per day ($)

vii) b : Set up cost per cycle ($)

viii) i_c: Average natural idle time cost per unit idle time ($).

ix) p_1 : Profit per unit item($)

x) p_0 : Purchasing price per unit item($)

xi) n+1 : The number of days per cycle, n is a positive integer.

xii) t : Cycle length (days)

xiii) T : Time horizon (days)

xiv) TAC : Total average inventory cost per cycle ($)

3. Model Formulation

The inventory starts at the beginning of the opening time duration t_1 and meets the demand quantity d per unit time. After that, it remains steady up to the closing time period t_2. Again it starts from the next day opening time and hence it continues up to $n+1$ days. Now we may classify the proposed model into three different cases:

Case-I: Model with demand rate exactly d

Case-II : Model with excess measured demand rate[$= d(1+\theta)$],

Case-III : Model with low measured demand rate[$= d(1-\theta)$],

The diagram of the proposed model is given below

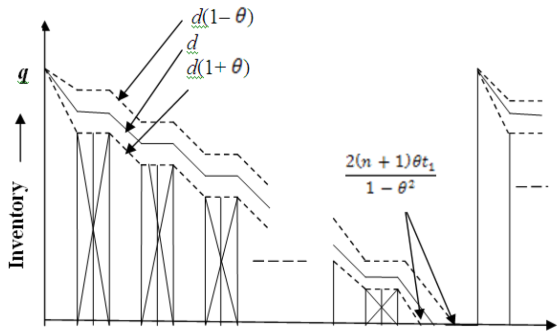


Fig. 1: EOQ Model

3.1 Model-I

When no measuring error occurs, the demand rate will be exactly d, therefore the inventory holding cost (HC) can be stated as

$$HC = h\left[\frac{1}{2}(2q - dt_1)t_1 + \frac{1}{2}(2q - 3dt_1)t_1 + \frac{1}{2}(2q - 5dt_1)t_1 + \cdots .n\ times + \frac{1}{2}d\ t_1^2\right]$$
$$+ h[(q - dt_1)t_2 + (q - 2dt_1)t_2 + (q - 3dt_1)t_2 + \cdots .n\ times]$$
$$= h\left[qt_1(1 + 1 + \cdots .n\ times) - \frac{1}{2}d\ t_1^2(1 + 3 + 5 + \cdots .2n - 1) + \frac{1}{2}d\ t_1^2\right]$$
$$+ h[q(1 + 1 + \cdots .n\ times)t_2 - d\ t_1t_2(1 + 3 + 5 + \cdots + n)]$$
$$= h\left[nqt_1 - \frac{1}{2}d\ t_1^2(n^2 - 1) + nqt_2 - n(n+1)dt_1t_2\right]$$

$$= \frac{(n+1)hdt_1}{2}[n(t_1+t_2)+t_1] \qquad \ldots(1)$$

Where $\quad q = (n+1)dt_1 \qquad \ldots(2)$

And $\quad t_1 + t_2 = 1 \qquad \ldots(3)$

Idle time cost $\quad PC = i_c(n+1)t_2 \qquad \ldots(4)$

Cycle time, $\quad t = (n+1)(t_1+t_2) \qquad \ldots(5)$

Now , using equations (1), (4) and (5) the total average inventory cost per cycle is given by

TAC =[Holding cost+ Idle time cost+ Setup cost]×Number of Cycles

$$=[HC + PC + b]\,\frac{T}{t}$$

$$= \left[\frac{(n+1)hdt_1}{2}\{n(t_1+t_2)+t_1\} + (n+1)i_c t_2 + b\right]\frac{T}{(n+1)(t_1+t_2)}$$

$$= \left[\frac{nhdt_1}{2} + \frac{hdt_1^2}{2(t_1+t_2)} + \frac{i_c t_2}{(t_1+t_2)} + \frac{b}{(n+1)(t_1+t_2)}\right]T \qquad \ldots(6)$$

3.2 Model-II

When the demand rate *d* is excessively measured, a positive error fraction θ of demand rate occurs, so the inventory holding cost (HC^-) can be calculated as follows

$$HC^- = h\left[\frac{1}{2}\{2q-d(1+\theta)t_1\}t_1 + \frac{1}{2}\{2q-3d(1+\theta)t_1\}t_1 + \frac{1}{2}\{2q-5d(1+\theta)t_1\}t_1 + \cdots .n\ times + \frac{(1-n\theta)t_1}{2(1+\theta)}\{q-nd(1+\theta)t_1\}\right]$$

$$+\, h[\{q-d(1+\theta)t_1\}t_2 + \{q-2d(1+\theta)t_1\}t_2 + \{q-3d(1+\theta)t_1\}t_2 \ldots\ldots n\ times]$$

$$= h\left[qt_1(1+1+\cdots .n\ times) - \frac{1}{2}d\,(1+\theta)t_1^2(1+3+5+\cdots .2n-1) + \frac{(1-n\theta)^2 d t_1^2}{2(1+\theta)}\right]$$

$$+h[q(1+1+\cdots .n\ times)t_2 - d\,(1+\theta)t_1t_2(1+3+5+\cdots+n)]$$

$$= h\left[nqt_1 - \frac{1}{2}d\,t_1^2\left\{n^2(1+\theta) - \frac{(1-n\theta)^2}{(1+\theta)}\right\} + nqt_2 - n(n+1)d(1+\theta)t_1t_2\right]$$

$$= \frac{(n+1)hdt_1}{2}[n(t_1+t_2)+t_1] - \frac{\theta(n+1)hdt_1}{2(1+\theta)}[(n+1)(t_1+t_2)-(1-n\theta)t_2]$$

using (2) [see (A.1)] (7)

Idle time cost = $\frac{(n+1)i_c}{(1+\theta)}\{t_2 + \theta(t_1+t_2)\}$ [see (A.2)] $\qquad \ldots(8)$

Loss due to excessive measure demand rate = $\frac{(p_0+p_1)(n+1)d\theta t_1}{(1+\theta)}$ [see (A.3)] (9)

Hence Total average inventory cost per cycle is given by

TAC^- = [Holding cost+ Loss due to excessive measure +Idle time cost+ Setup cost]×Number of Cycles

$$TAC^- = \left[\frac{(n+1)hdt_1}{2}\{n(t_1+t_2)+t_1\}+(n+1)i_c t_2+b\right]\frac{T}{(n+1)(t_1+t_2)}$$
$$+\frac{\theta t_1 T}{1+\theta}\left[\frac{i_c+(p_0+p_1)d+0.5hd(1-n\theta)t_2}{(t_1+t_2)}-\frac{(n+1)h\,d}{2}\right]$$
$$= TAC+\frac{\theta t_1 T}{1+\theta}\left[\frac{i_c+(p_0+p_1)d+0.5hd(1-n\theta)t_2}{(t_1+t_2)}-\frac{(n+1)h\,d}{2}\right] \quad \ldots(10)$$

3.3 Model-III

When the demand rate d is measured low, a negative error fraction θ of demand rate occurs for which we may define the inventory holding cost (HC^+) as

$$HC^+ = h\left[\frac{1}{2}\{2q-d(1-\theta)t_1\}t_1+\frac{1}{2}\{2q-3d(1-\theta)t_1\}t_1+\frac{1}{2}\{2q-5d(1-\theta)t_1\}t_1 + \cdots .n\ times+\frac{(1+n\theta)t_1}{2(1-\theta)}\{q-nd(1-\theta)t_1\}\right]$$
$$+\ h[\{q-d(1-\theta)t_1\}t_2+\{q-2d(1-\theta)t_1\}t_2+\{q-3d(1-\theta)t_1\}t_2 \ldots\ldots n\ times]$$
$$= h\left[qt_1(1+1+\cdots .n\ times)-\frac{1}{2}d\ (1-\theta)t_1^2(1+3+5+\cdots .2n-1)+\frac{(1+n\theta)^2 d\,t_1^2}{2(1-\theta)}\right]$$
$$+h[q(1+1+\cdots .n\ times)t_2-d\ (1-\theta)t_1t_2(1+3+5+\cdots+n)]$$
$$= h\left[nqt_1-\frac{1}{2}d\,t_1^2\left\{n^2(1-\theta)-\frac{(1+n\theta)^2}{(1-\theta)}\right\}+nqt_2-n(n+1)d(1-\theta)t_1t_2\right]$$
$$= \frac{(n+1)hdt_1}{2}[n(t_1+t_2)+t_1]+\frac{\theta(n+1)hdt_1}{2(1-\theta)}[(n+1)(t_1+t_2)-(1+n\theta)t_2]$$

using (2) [See (A.4)] …(11)

Idle time cost = $\frac{(n+1)i_c}{(1-\theta)}\{t_2-\theta(t_1+t_2)\}$ [see (A.5)] …(12)

Profit gained for low measure = $\frac{p_1(n+1)d\theta t_1}{(1-\theta)}$ [see (A.6)] …(13)

Hence Total average inventory cost is given by

TAC^+ = [Holding cost-Profit gained for low measure +Idle time cost+ Setup cost]×Number of Cycles

$$TAC^+ = \left[\frac{(n+1)hdt_1}{2}\{n(t_1+t_2)+t_1\}+(n+1)i_c t_2 + b\right]\frac{T}{(n+1)(t_1+t_2)}$$

$$+\frac{\theta t_1 T}{1-\theta}\left[\frac{0.5hd(1+n\theta)t_2 - p_1 d - i_c}{(t_1+t_2)}+\frac{(n+1)h\,d}{2}\right]$$

$$= TAC + \frac{\theta t_1 T}{1-\theta}\left[\frac{0.5hd(1+n\theta)t_2 - p_1 d - i_c}{(t_1+t_2)}+\frac{(n+1)h\,d}{2}\right] \quad \ldots(14)$$

Now our problem (6) , (10) and (14) can be stated as

$$Minimize\ TAC = \left[\frac{nhdt_1}{2}+\frac{hdt_1^2}{2(t_1+t_2)}+\frac{i_c t_2}{(t_1+t_2)}+\frac{b}{(n+1)(t_1+t_2)}\right]T \quad \ldots(15)$$

$$Minimize\ \ TAC^- = TAC + \frac{\theta t_1 T}{1+\theta}\left[\frac{i_c+(p_0+p_1)d+0.5hd(1-n\theta)t_2}{(t_1+t_2)}-\frac{(n+1)h\,d}{2}\right] \quad \ldots(16)$$

$$Minimize\ \ TAC^+ = TAC + \frac{\theta t_1 T}{1-\theta}\left[\frac{0.5hd(1+n\theta)t_2 - p_1 d - i_c}{(t_1+t_2)}+\frac{(n+1)h\,d}{2}\right] \quad \ldots(17)$$

Subject to the conditions (2), (3) and (5).

Note: **Special Cases**

Case-I: If we take $t_2 \to 0$ then $i_c = 0.0$, so the equation (15) reduces to

$$TAC = \left[\frac{hq}{2}+\frac{bd}{q}\right]T$$ This is the classical EOQ model. …(18)

Case-II: If $\theta \to 0$ then $p_1 \to 0, p_0 \to 0$ and so

$$TAC^- = TAC = TAC^+ \quad \ldots(19)$$

Now to obtain the optimum number of n , we may design the following algorithm.

4. Computational Algorithm-1

(1) Set $n = 2$

(2) Compute *TAC*, TAC^- and TAC^+ from (15), (16) and (17) respectively and find t_1 , check whether $t_1 < 1$ and $TAC\ (n) < TAC(n+1)\ and\ TAC\ (n-1) > TAC(n)$ etc. in each of the case otherwise set $n = n + 1$ go to (2)

(3) Calculate TAC, TAC^- and TAC^+ from (15), (16) and (17) respectively and find corresponding t_1, t_2 , t and q using (2), (3) and (5) and get the optimum result.

(4) Stop

4.1 Numerical Example-1

Let in a tourist spot, a fruits seller purchasing a certain fruit with cost $p_0 =$ \$ 2.5 per 10 kilogram and began to sale with a profit $p_1 =$ \$ 0.38 per 10 kilogram. If (s)he has a setup cost per cycle b = \$ 100, demand quantity per day d = 20 kilogram , holding cost h = \$1.5 per 10 kilogram per day , idle time cost per day $i_c =$ \$7.5 , measuring error $\theta =$0.02 times of the demand quantity per day, then for a time horizon T = 100 days , using equations (15)-(17) the average minimum cost can be obtained as follows:

Table-1

Model	n	t_1^*	t_2^*	t^*	q^* (×10 kg.)	Z^* (\$)
I	4	0.500	0.500	5	5.000	2712.500
II	4	0.462	0.538	5	4.617	2718.607
III	4	0.505	0.495	5	5.051	2712.547

4.2 Numerical Example-2

For the same data set as taken in example-1, if we take the opening time = closing time =1/2 day then the optimal results can be obtained as follows:

Table-2

Model	n	t_1^*	t_2^*	t^*	q^* (×10 kg.)	Z^* (\$)
I	11	0.5	0.5	12	12	2070.833
II	11	0.5	0.5	12	12	2066.760
III	10	0.5	0.5	11	11	2080.917

5. Fuzzy Mathematical Model

In the crisp mathematical model we have optimized the objective functions considering the opening and closing time period as crisp number. But in reality, they are very much flexible in nature, so we may assume them as fuzzy number. However, the above optimum Table-1 shows that if the opening time duration t_1 is less than 12 hours and closing time duration t_2 is greater than or equal to 12 hours in a day then the total average inventory cost is minimum. For this reason let us consider t_1 as *L*-fuzzy and t_2 as *R*-fuzzy number. The membership functions of t_1, t_2 and objective functions are given below:

$$\mu_{t_{1}i}(x_i) = \begin{cases} 1 & for \;\; x_i > t_{10\,i} \\ \frac{t_{10i}-x_i}{p_{1i}} & for \;\; t_{10i} - p_{1i} \le x_i \le t_{10i} \\ 0 & for \;\; x_i < t_{10i} - p_{1i} \end{cases} \qquad \ldots(20)$$

$$\mu_{t_{2}i}(x_i) = \begin{cases} 0 & for \;\; x_i < t_{20i} \\ \frac{x_i - t_{20i}}{p_{2i}} & for \;\; t_{20i} \le x_i \le t_{20i} + p_{2i} \\ 1 & for \;\; x_i > t_{20i} + p_{2i} \end{cases} \qquad \ldots(21)$$

$$\mu_{z\,i}(z_i) = \begin{cases} 0 & for \;\; z_i < z_{0i} \\ \frac{z_i - z_{0i}}{p_{0i}} & for \;\; z_{0i} \le z_i \le z_{0i} + p_{0i} \\ 1 & for \;\; z_i > z_{0i} + p_{0i} \end{cases} \quad \text{for } i = 1,2,3 \qquad \ldots(22)$$

Now, the fuzzy mathematical problems are given by

$$Minimize\ \tilde{z}_1 = \left[\frac{nhd\tilde{t}_1}{2} + \frac{hd\,\tilde{t}_1^2}{2(\tilde{t}_1 + \tilde{t}_2)} + \frac{i_c\tilde{t}_2}{(\tilde{t}_1 + \tilde{t}_2)} + \frac{b}{(n+1)(\tilde{t}_1 + \tilde{t}_2)}\right] T \qquad \ldots(23)$$

$$Minimize\ \tilde{z}_2 = \tilde{z}_1 + \frac{\theta\tilde{t}_1 T}{1+\theta}\left[\frac{i_c + (p_0 + p_1)d + 0.5hd(1-n\theta)\tilde{t}_2}{(\tilde{t}_1 + \tilde{t}_2)} - \frac{(n+1)h\,d}{2}\right] \qquad \ldots(24)$$

$$Minimize\ \tilde{z}_3 = \tilde{z}_1 + \frac{\theta\tilde{t}_1 T}{1-\theta}\left[\frac{0.5hd(1+n\theta)\tilde{t}_2 - p_1 d - i_c}{(\tilde{t}_1 + \tilde{t}_2)} + \frac{(n+1)h\,d}{2}\right] \qquad \ldots(25)$$

Subject to the conditions (2) , (5) and $\tilde{t}_1 + \tilde{t}_2 \cong 1$ … (26)

Now using aspiration level α to each membership function, the equivalent crisp optimization problems can be obtained with the help of Bellman and Zadeh (1970) and Zimmermann (1978). Therefore, applying the membership functions (20)-(22) , the fuzzy non-linear equations (23)-(25) may be transformed into crisp equivalent and they are as follows:

$$Max\ \alpha_k$$

$$Subject\ to\ \ z_k \le z_{0k} - \alpha_k p_{0k} \qquad \ldots(27)$$

$$z_1 = \left[\frac{nhd(t_{101}-\alpha_1 p_{11})}{2} + \frac{0.5hd\,(t_{101}-\alpha_1 p_{11})^2 + i_c(t_{201}+\alpha_1 p_{21}) + b/(n+1)}{t_{101}+t_{201}-\alpha_1(p_{11}-p_{21})}\right] T \quad \ldots (28)$$

$$z_2 = \frac{\theta(t_{102}-\alpha_2 p_{12})T}{1+\theta}\left[\frac{i_c + (p_0+p_1)d + 0.5hd(1-n\theta)(t_{202}+\alpha_2 p_{22})}{t_{102}+t_{202}-\alpha_2(p_{12}-p_{22})} - \frac{(n+1)h\,d}{2}\right]$$
$$+\left[\frac{nhd(t_{102}-\alpha_2 p_{12})}{2} + \frac{0.5hd\,(t_{102}-\alpha_2 p_{12})^2 + i_c(t_{202}+\alpha_2 p_{22}) + b/(n+1)}{t_{102}+t_{202}-\alpha_2(p_{11}-p_{22})}\right] T \quad \ldots (29)$$

$$z_3 = \frac{\theta(t_{103}-\alpha_3 p_{13})T}{1-\theta}\left[\frac{0.5hd(1+n\theta)(t_{203}+\alpha_3 p_{23}) - p_1 d - i_c}{t_{103}+t_{203}-\alpha_3(p_{13}-p_{23})} + \frac{(n+1)h\,d}{2}\right]$$
$$+\left[\frac{nhd(t_{103}-\alpha_3 p_{13})}{2} + \frac{0.5hd\,(t_{103}-\alpha_3 p_{13})^2 + i_c(t_{203}+\alpha_3 p_{23}) + b/(n+1)}{t_{103}+t_{203}-\alpha_3(p_{13}-p_{23})}\right] T \quad \ldots (30)$$

and $t_{1k} \le t_{10k} - \alpha_k p_{1k}$, $t_{2k} \ge t_{20k} + \alpha_k p_{2k}$ for k = 1,2 and 3 (31)

5.1 Formulation of Intuitionistic Fuzzy Optimization (IFO) technique

We know that, IFS is applied when the sufficient information in the fuzzy sets is not available or a lack of knowledge in fuzzy decision making process exists. In such cases the IFS is a proper subset of the Fuzzy set for which the degree of rejection (non-membership) and the degree of acceptance (membership) are defined simultaneously and they are not complementary to each other. In fuzzy set, our aim is to maximize the support (degree of acceptance) of the membership functions so consequently, minimize the height of negation or non-membership (degree of rejection). Thus we may transform our optimization problem into the following way:

$$Max \quad \mu_i(\bar{X}) \quad \forall\, \bar{X} \in R$$
$$Min \quad \vartheta_i(\bar{X}) \quad \forall\, \bar{X} \in R$$
$$\vartheta_i(\bar{X}) \ge 0$$

Subject to $\mu_i(\bar{X}) \ge \vartheta_i(\bar{X})$

$$\mu_i(\bar{X}) + \vartheta_i(\bar{X}) \le 1, \bar{X} \ge 0 \text{ for } i = 1,2 \text{ and } 3 \quad \ldots (32)$$

Where $\mu_i(\bar{X})$ denotes the degree of membership (acceptance) function of $\bar{X}$ to the i^{th} IF sets and $\vartheta_i(\bar{X})$ denotes the degree of non-membership (rejection) function of $\bar{X}$ for the i^{th} IF sets.

5.2 IFO technique for solving the objective functions with linear membership and non-membership functions

Here we have to define the membership functions of the objective functions with their lower and upper bounds. Let us use L_k^{acc} as lower and U_k^{acc} as upper bound of the k th objective functions. These values are determined as follows: The lower and upper bounds of the opening time duration t_1 and closing time duration t_2 are taken first. These predictions depend upon the decision maker's choice and expertise that is concerned with the insight of the model. These values are then arranged as a pair like (t_1, t_2) = { (max, max), (max , min), (min , max), (min, min)}. The optimum value of each objective function for each pair (t_1, t_2) for n =1, 2, 3, 4, … is found out. Let $\bar{X}_{1k}^*$, $\bar{X}_{2k}^*$, $\bar{X}_{3k}^*$…….. $\bar{X}_{jk}^*$ be the respective optimal solutions to the k th objective function $z_k(\bar{X}_{jk})$ for j =1,2,3 and 4 , k =1,2 and 3 respectively.

For each objective the lower bound $L_k^{acc} = Min\{z_k(\bar{X}_{jk})\}$ and upper bound $U_k^{acc} = Max\{z_k(\bar{X}_{jk})\}$. But in Intuitionistic Fuzzy Optimization (IFO), the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To obtain the membership functions under IFO environment let L_k^{rej} be the lower and U_k^{rej} be the upper bound of the objective functions $z_k(\bar{X}_{jk})$ where $L_k^{acc} \le L_k^{rej} \le U_k^{rej} \le U_k^{acc}$.These values are obtained from the following definition.

Definition-1: For minimization problem, the lower bound for non-membership function (rejection) is always greater than that of the membership function (acceptance).

We take lower bound and upper bound for non-membership function as follows:

$$\begin{cases} L_k^{rej} = L_k^{acc} + \delta(U_k^{acc} - L_k^{acc}) \ \ for\ 0 < \delta < 1 \\ U_k^{rej} = U_k^{acc} + \delta(U_k^{acc} - L_k^{acc}) \ \ for\ \delta = 0 \end{cases} \quad \dots (33)$$

The linear membership function and linear non- membership function for the objective $z_k(\bar{X})$ are given by

$$\mu_k(z_k(\bar{X})) = \begin{cases} 1 & for \;\; z_k(\bar{X}) \le L_k^{acc} \\ \frac{U_k^{acc} - z_k(\bar{X})}{U_k^{acc} - L_k^{acc}} & for \;\; L_k^{acc} \le z_k(\bar{X}) \le U_k^{acc} \\ 0 & for \;\; z_k(\bar{X}) \ge U_k^{acc} \end{cases} \quad \ldots (34)$$

$$\vartheta_k(z_k(\bar{X})) = \begin{cases} 1 & for \;\; z_k(\bar{X}) \ge U_k^{rej} \\ \frac{z_k(\bar{X}) - L_k^{rej}}{U_k^{rej} - L_k^{rej}} & for \;\; L_k^{rej} \le z_k(\bar{X}) \le U_k^{rej} \\ 0 & for \;\; z_k(\bar{X}) \le L_k^{rej} \end{cases} \quad \ldots (35)$$

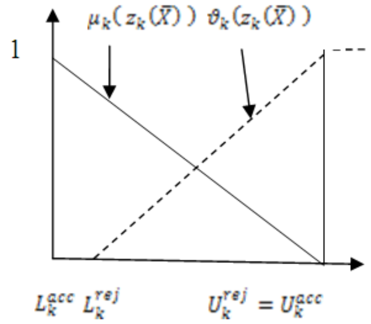


Fig-2: (Non) Membership function for $z_k(\bar{X})$

The linear membership and non-membership function of t_1 are given by

$$\mu_{t_{1k}}(x_k) = \begin{cases} 0 & for \;\; x_k > t_{1Uk}^{acc} \\ \frac{t_{1Uk}^{acc} - x_k}{t_{1Uk}^{acc} - t_{1Lk}^{acc}} & for \;\; t_{1Lk}^{acc} \le x_k \le t_{1Uk}^{acc} \\ 1 & for \;\; x_k < t_{1Lk}^{acc} \end{cases} \quad \ldots (36)$$

$$\vartheta_{t_{1k}}(x_k) = \begin{cases} 1 & for \;\; x_k > t_{1Uk}^{rej} \\ \frac{x_k - t_{1Lk}^{rej}}{t_{1Uk}^{rej} - t_{1Lk}^{rej}} & for \;\; t_{1Lk}^{rej} \le x_k \le t_{1Uk}^{rej} \\ 0 & for \;\; x_k < t_{1Lk}^{rej} \end{cases} \quad \ldots (37)$$

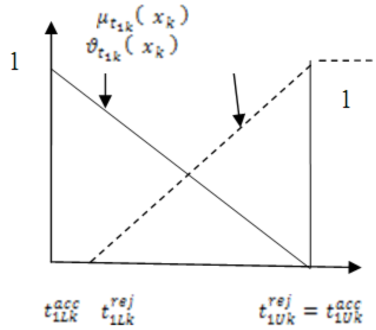


Fig-3: (Non)membership function for t_{1k} (L-fuzzy)

The linear membership and non-membership function of t_2 are given by

$$\mu_{t_{2k}}(x_k) = \begin{cases} 1 & for \;\; x_k > t_{2Uk}^{acc} \\ \frac{x_k - t_{2Lk}^{acc}}{t_{2Uk}^{acc} - t_{2Lk}^{acc}} & for \;\; t_{2Lk}^{acc} \le x_k \le t_{2Uk}^{acc} \\ 0 & for \;\; x_k < t_{2Lk}^{acc} \end{cases} \quad \text{for } k = 1,2 \text{ and } 3 \qquad \ldots(38)$$

$$\vartheta_{t_{2k}}(x_k) = \begin{cases} 0 & for \;\; x_k > t_{2Uk}^{rej} \\ \frac{t_{2Uk}^{rej} - x_k}{t_{2Uk}^{rej} - t_{2Lk}^{rej}} & for \;\; t_{2Lk}^{rej} \le x_k \le t_{2Uk}^{rej} \\ 1 & for \;\; x_k < t_{2Lk}^{rej} \end{cases} \quad \text{for } k = 1,2 \text{ and } 3 \qquad \ldots(39)$$

The notations keep their usual meanings.

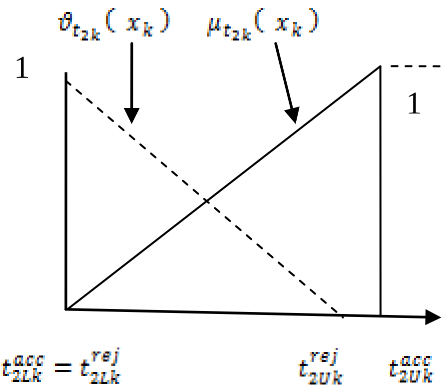


Fig-4:(Non) Membership function for t_{2k}(*R*-fuzzy

Now the fuzzy decision of Bellman and Zadeh (1970) together with membership and Non-membership functions of (34)-(39) , the IFO problem can be written as

$$Max \;\; \mu_k(\bar{X}) \;\; \forall \bar{X} \in R$$

$$Min\ \vartheta_k(\bar{X})\ \forall \bar{X} \in R$$

$$\vartheta_k(\bar{X}) \geq 0$$

Subject to $\mu_k(\bar{X}) \geq \vartheta_k(\bar{X})$

$\mu_k(\bar{X}) + \vartheta_k(\bar{X}) \leq 1, \bar{X} \geq 0$ for k =1,2 and 3 … (40)

The above problem can be reduced following Angelov (1997) to the following form

$$\begin{cases} Max\ \alpha - \beta \\ Subject\ to\ \ z_k(\bar{X}) \leq U_k^{acc} - \alpha(U_k^{acc} - L_k^{acc}) \\ z_k(\bar{X}) \geq L_k^{rej} + \beta(U_k^{rej} - L_k^{rej}) \\ \beta \geq 0, \alpha \geq \beta,\ \alpha + \beta \leq 1, \bar{X} \geq 0 \end{cases}$$ for k =1,2 and 3 … (41)

5.3 Computational Algorithm-2

Step-1. Optimize the objective function for different fixed pair of values of (t_1, t_2) for n = 1, 2, 3.. Find the lower bound and upper bound of the objective functions as

$L_k^{acc} = Min\{z_k(\bar{X}_{jk})\}$ and upper bound $U_k^{acc} = Max\{z_k(\bar{X}_{jk})\}$ for k = 1,2 ,3 and j = 1,2,3 and 4 respectively.

Step-2. At first construct the membership functions of objective goals, opening time duration t_1 and closing time duration t_2 with appropriate tolerances (decision maker's choice) . Then from (33) , for suitable choice of δ (requires decision maker's expertise and knowledge) find the non-membership functions of the same.[in case of t_2, the formulae stated in (33) for acceptance and rejections will be interchanged]

Step-3. Construct the fuzzy programming problem and its equivalent crisp optimization problem stated in (41).

Step-4. Use a compromised/ ideal solution of t_1 and t_2 for the problem stated in (41). Solve this non-linear programming problem with appropriate programming for n = 1, 2, 3, …..for the decision variables α and β and then obtain t_1^*, t_2^*, q^* and z^*

Note: Compromise solution of t_1 and t_2 are given by:

$$\begin{cases} t_1^* = 0.5\left(t_{1Uk}^{acc} + t_{1Lk}^{rej}\right) + 0.5\beta^*\left(t_{1Uk}^{rej} - t_{1Lk}^{rej}\right) - 0.5\alpha^*\left(t_{1Uk}^{acc} - t_{1Lk}^{acc}\right) \\ t_2^* = 0.5\left(t_{2Uk}^{acc} + t_{2Lk}^{rej}\right) - 0.5\beta^*\left(t_{2Uk}^{rej} - t_{2Lk}^{rej}\right) + 0.5\alpha^*\left(t_{2Uk}^{acc} - t_{2Lk}^{acc}\right) \end{cases}$$ … (42)

5.4 Numerical Example-3

Let us take, purchasing price p_0 = \$ 2.5 per 10 kilogram, profit p_1 = \$ 0.38 per 10 kilogram, setup cost per cycle b = \$ 100, demand quantity per day d = 20 kilogram, holding cost h = \$1.5 per 10 kilogram per day , idle time cost per day i_c = \$7.5 , measuring error θ =0.02 times of the demand quantity per day, then for a time horizon T = 100 days and the following several initial values of different parameters with their corresponding tolerances $t_{101} = t_{102} = t_{103} = t_{201} = t_{202} = t_{203} = 0.5$, $p_{01} = p_{02} = p_{03} = 150$ $p_{11} = p_{12} = p_{13} = p_{21} = p_{22} = p_{23} = .03$, $z_{01} = z_{02} = z_{03} = 2700$ for fuzzy models and for IFO problems we may take

$t_{1U1}^{acc} = t_{1U1}^{rej} = 0.5, t_{1L1}^{acc}$=0.47,

$t_{1L1}^{rej} = 0.48, t_{2L1}^{acc} = t_{2L1}^{rej} = 0.5$, $t_{2U1}^{acc} = 0.53, t_{2U1}^{rej} = 0.52$

, $L_1^{acc} = 2020.0,\ L_1^{rej} = 2030.0$, $U_1^{acc} = U_1^{rej}$ =2090.0 , $t_{1U2}^{acc} = t_{1U2}^{rej} = 0.47, t_{1L2}^{acc}$=0.44,

$t_{1L2}^{rej} = 0.45, t_{2L2}^{acc} = t_{2L2}^{rej} = 0.53$, $t_{2U2}^{acc} = 0.56, t_{2U2}^{rej} = 0.55$

, $L_2^{acc} = 2135.0,\ L_2^{rej} = 2155.0$, $U_2^{acc} = U_2^{rej}$ =2210.0

$t_{1U3}^{acc} = t_{1U3}^{rej} = 0.44, t_{1L3}^{acc}$=0.41,

$t_{1L3}^{rej} = 0.42, t_{2L3}^{acc} = t_{2L3}^{rej} = 0.56$, $t_{2U3}^{acc} = 0.59, t_{2U3}^{rej} = 0.58$

$L_3^{acc} = 1980.0,\ L_3^{rej} = 1995.0$, $U_3^{acc} = U_3^{rej}$ =2040.0

For the solutions of equations (27)-(31) and (41)-(42) and get the following results.

Table-3. Solution for GFNLP & IFO

Model	Type	n	t_1^*	t_2^*	f^*	q^* (×10 kg.)	z^* (\$)	α	β
I	GFNLP	4	0.497	0 .503	5	4.975	2712.501	.917	….
	IFO	11	0.477	0.523	12	11.455	2047.052	.614	.284
II	GFNLP	4	0.496	0.504	5	4.962	2718.783	.875	….
	IFO	7	0.445	0.555	8	7.126	2164.653	.605	.176
III	GFNLP	4	0.497	0.503	5	4.975	2712.555	.916	….
	IFO	10	0.418	0.582	11	9.202	2006.949	.551	.239

From the above Table-3 we see that, the solution obtained from IFO technique gives the better result than GFNLP technique for each model. The empirical data shows, in IFO policy, model with low measured demand rate (degree of rejection 0.239) has a minimum optimum inventory cost than the model with excessively measured demand rate (degree of rejection 0.176) and actual demand rate.

5.5 Sensitivity Analysis

From the Table-1, the deterministic model-II shows the maximum inventory cost relative to the model-I and model-III. So taking the same data set as per example-1, we may consider the range of variations of the parameters { *h*, *d*, i_c, p_0, p_1, θ, *b* }

from -50% to + 50% and have the following optimal Table-4

Table-4. Sensitivity analysis for model –II

Parameter	% change	n	t_1^*	t_2^*	q^* (×10 kg.)	z^* ($)	$\frac{q^*-q_*}{q_*}\times 100\%$	$\frac{z^*-z_*}{z_*}\times 100\%$
	-50	9	.423	.577	8.470	1736.766	83.45	-36.11
	-30	6	.517	.483	7.231	2151.040	56.61	-20.87
h	+30	3	.394	.606	3.149	3220.337	-31.79	18.46
	+50…	…..	….	…...	……	……	…….	
	-50	9	.462	.538	4.617	1734.267	0.0	-36.20
	-30	6	.533	.467	5.222	2149.262	13.10	-20.94
d	+30	3	.385	.615	4.002	3221.656	-13.13	18.50
	+50	…	…..	….	……	……..	…..	…….
	-50	..	…..	……	…….	…….	…….	……
	-30	2	.711	.289	4.268	3783.842	-7.55	39.18
i_c	+30	6	.212	.788	2.972	2396.928	-35.62	-11.83
	+50	7	.212	.788	3.400	2368.347	-26.35	-12.88
	-50	4	.478	.522	4.783	2716.303	3.59	-0.08
p_0	-30	4	.472	.528	4.716	2717.235	2.14	-0.05
	+30	4	.452	.548	4.517	2719.950	-2.16	0.04
	+50	4	.445	.555	4.450	2720.829	-3.62	0.08
	-50	4	.464	.536	4.642	2718.262	0.54	-0.01
p_1	-30	4	.463	.537	4.632	2718.400	0.32	-0.01
	+30	4	.460	.540	4.601	2718.813	-0.35	0.01

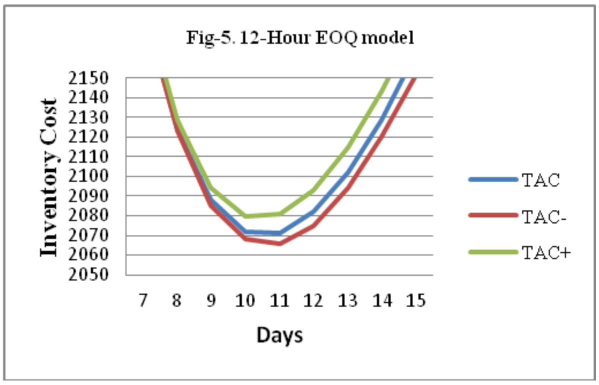
	+50	4	.459	.541	4.591	2718.950	-0.56	0.01
	-50	4	.480	.520	4.808	2715.653	4.10	-0.01
θ	-30	4	.473	.527	4.731	2716.858	2.46	-0.06
	+30	4	.450	.550	4.502	2720.286	-2.49	0.06
	+50	4	.443	.557	4.426	2721.368	-4.13	0.01
	-50	4	.462	.538	4.617	1718.607	0.0	-36.78
b	-30	4	.462	.538	4.617	2118.607	0.0	-22.07
	+30	4	.462	.538	4.617	3318.607	0.0	22.07
	+50	4	.462	.538	4.617	3718.607	0.0	36.78

6. Comments on Sensitivity Analysis

The optimal solution for z (*n*, t_1) is based on the fixed set of parameters { *h*, *d*, i_c, p_0 p_1, θ, *b* }. Table-4 shows a sensitivity analysis for the optimal values of cycle time *n*+1 days, Optimum opening time duration t_1, optimum closing time duration t_2, optimum lot size *q* and the cost function z. We see, *q* is insensitive with the change of ordering cost *b* from -50% to +50% *viz*, cost function is highly sensitive. The order quantity *q* is moderately sensitive with change of demand rate *d* from –50% to +50% , at –30% change, *q* increased by 13.10 %, *viz.* at –50% change the average cost reduces to 36.20 %. For the changes from –50% to +50% of natural idle time cost i_c, the order quantity *q* and average cost function z have moderately unidirectional and opposite directional change respectively. The cost function is insensitive for the changes of the parameters p_0 p_1, and θ, Finally, the change of holding cost *h* gives a very high degree of the opposite directional change in the order quantity *q* and for the average cost function *z* , its change is unidirectional on average.Throughout the whole table we see that, for 50% decrease of set up cost *b*, the value of the opening time period t_1 *i,e.* **0.462** day or in other words **11-hrs.05-mins. 17-s** makes a minimum total average inventory cost $ **1718.61**.

7. Graphical Interpretations of the Model

We may use Excel graphical format to have the behavioral curves of the average cost functions z_1, z_2, and z_3 at 12-hour inventory opening time period with respect to different days. From the Fig.5, we see that for the model with excessively measured demand rate (model-II) giving minimum total average inventory cost $ **2066.76** for **12** days optimum cycle time with 12-hours inventory opening time period. Astonishing to observe that, for low measured demand rate model-III gives relatively high inventory cost $ **2080.92** at **11** days optimal cycle time.



8. Conclusion

In this paper, a new concept concerning the opening time duration in a day in a simple classical EOQ model has been discussed. The major difference of the model lies in its conventional literature. Another major concept is underlying in its incorrectness of supplied demand rate. However, as no research works have been done along this direction, so the model tunes a new concept as well. In another part, for practical sense, everything is uncertain or more specific time is non-randomly uncertain and it is flexible in nature. For this reason a fuzzy time variable has been introduced into the model to cope with logical studies. There are many optimization techniques available in the studies of Operations Research, among them the most common and reliable method named IFO technique has been used to illustrate the model. Optimization is made with the help of LINGO software. The beauty of the model is that, a nice result of inventory run time per day has been reached and it is nearly 12-hours which is definitely a new achievement in present research.

9. Scope of Future Work

In this paper natural leisure time is considered in the intermediate cycle time, incorrect out-flow of demand rate per unit time has also been taken care of. Incorporating these phenomena in inventory models several deterministic, fuzzy and fuzzy –stochastic models can be developed. Genetic algorithm, fuzzy geometric programming, fuzzy stochastic programming etc. can also be applied to obtain the better improved solution.

Acknowledgement

The author is indebted to the anonymous referees for their constructive comments and suggestions.

Appendix

1. Holding cost for **Model-II [** for demand d (1+ θ)] can be calculated as follows:

   h× Area of (the n vertical trapeziums of width t_1 each +the triangle with base t' and height $q - n\,d\,(1+\theta)\,t_1$ + the n vertical rectangles of width t_2 each)

$$HC^- = \left[\frac{1}{2}\{2q - d(1+\theta)t_1\}t_1 + \frac{1}{2}\{2q - 3d(1+\theta)t_1\}t_1 + \frac{1}{2}\{2q - 5d(1+\theta)t_1\}t_1 + \cdots .n\ times + \frac{1}{2}t'\{q - nd(1+\theta)t_1\}\right]$$

$$+ h[\{q - d(1+\theta)t_1\}t_2 + \{q - 2d(1+\theta)t_1\}t_2 + \{q - 3d(1+\theta)t_1\}t_2 \ldots\ldots n\ times]$$

Here t' can be obtained as $d(1+\theta)t' = q - nd(1+\theta)t_1 \Rightarrow t' = \frac{t_1(1-n\theta)}{1+\theta}$

$since, q = (n+1)\,dt_1$

Therefore,we have the required holding cost

$$HC^- = h\left[qt_1(1+1+\cdots .n\ times) - \frac{1}{2}d\,(1+\theta)t_1^2(1+3+5+\cdots .2n-1) + \frac{(1-n\theta)^2 d\,t_1^2}{2(1+\theta)}\right]$$

$$+ h[q(1+1+\cdots .n\ times)t_2 - d\,(1+\theta)t_1t_2(1+3+5+\cdots+n)]$$

$$= h\left[nqt_1 - \frac{1}{2}d\,t_1^2\left\{n^2(1+\theta) - \frac{(1-n\theta)^2}{(1+\theta)}\right\} + nqt_2 - n(n+1)d(1+\theta)t_1t_2\right]$$

$$= \frac{(n+1)hdt_1}{2}[n(t_1+t_2)+t_1] - \frac{\theta(n+1)hdt_1}{2(1+\theta)}[(n+1)(t_1+t_2) - (1-n\theta)t_2]$$

using (2) (A.1)

Idle time can be calculated as follows :

2. Total idle time

$$= (n+1)t_2 + t_1 - t' = (n+1)t_2 + t_1 - \frac{t_1(1-n\theta)}{1+\theta} = \frac{(n+1)}{(1+\theta)}\{t_2 + \theta(t_1+t_2)\}$$

So, the idle time cost = $\frac{(n+1)i_c}{(1+\theta)}\{t_2 + \theta(t_1+t_2)\}$ (A.2)

3. Excessive demand quantity

$$= n\theta d t_1 + \theta\, d\, t' = n\theta d t_1 + \frac{t_1(1-n\theta)}{1+\theta} d t_1 = \frac{\theta(n+1)d t_1}{1+\theta}$$

Loss due to excessive measure demand rate $= \frac{(p_0+p_1)(n+1)d\theta t_1}{(1+\theta)}$ (A.3)

4. Holding cost for **Model-III [** for demand $d\,(1-\theta)$)] can be calculated as follows:

$h\times$ Area of (the n vertical trapeziums of width t_1 each +the triangle with base t'' and height $q - n\,d\,(1-\theta)\,t_1$+ the n vertical rectangles of width t_2 each)

$$HC^+ = h\Big[\tfrac{1}{2}\{2q - d(1-\theta)t_1\}t_1 + \tfrac{1}{2}\{2q - 3d(1-\theta)t_1\}t_1 + \tfrac{1}{2}\{2q - 5d(1-\theta)t_1\}t_1 + \cdots .n\,times + \tfrac{t''}{2}\{q - nd(1-\theta)t_1\}\Big]$$
$$+\, h[\{q - d(1-\theta)t_1\}t_2 + \{q - 2d(1-\theta)t_1\}t_2 + \{q - 3d(1-\theta)t_1\}t_2 \ldots\ldots n\,times]$$

Where t'' can be obtained from $d(1-\theta)t'' = q - nd(1-\theta)t_1$

$$\Rightarrow t'' = \frac{t_1(1+n\theta)}{1-\theta} \quad since, q = (n+1)\,dt_1$$

Therefore,

$$HC^+ = h\left[q t_1(1+1+\cdots .n\,times) - \tfrac{1}{2}d\,(1-\theta)t_1^2(1+3+5+\cdots .2n-1) + \frac{(1+n\theta)^2 d\, t_1^2}{2(1-\theta)}\right]$$

$$+h[q(1+1+\cdots .n\,times)t_2 - d\,(1-\theta)t_1 t_2(1+3+5+\cdots+n)]$$

$$= h\left[nq t_1 - \frac{1}{2}d\, t_1^2\left\{n^2(1-\theta) - \frac{(1+n\theta)^2}{(1-\theta)}\right\} + nq t_2 - n(n+1)d\,(1-\theta)t_1 t_2\right]$$

$$= \frac{(n+1)hdt_1}{2}[n(t_1+t_2)+t_1] + \frac{\theta(n+1)hdt_1}{2(1-\theta)}[(n+1)(t_1+t_2) - (1+n\theta)t_2] \quad \text{(A.4)}$$

5. The total idle time $= n\; t_2 + t_1 + t_2 - t'' = (n+1)t_2 + t_1 - \frac{t_1(1+n\theta)}{1-\theta} = \frac{(n+1)}{(1-\theta)}\{t_2 - \theta(t_1+t_2)\}$

Idle time cost $= \frac{(n+1)i_c}{(1-\theta)}\{t_2 - \theta(t_1+t_2)\}$ (A.5)

6. The surplus demand quantity = $n\theta\, d\, t_1 + \theta d t'' = \theta d\left[n t_1 + \frac{t_1(1+n\theta)}{1-\theta}\right] = \frac{(n+1)d\theta t_1}{(1-\theta)}$

Profit gained for low measure = $\frac{p_1(n+1)d\theta t_1}{(1-\theta)}$ (A.6)

References

[1] A. Kaufmann, M. Gupta, Fuzzy Mathematical Models in Engineering and Management Science, North Holland**,** 1988.

[2] Alireza Kabiran, Sigurdur Olafsson, Continuous Optimization via Simulation using Golden Region Search , European Journal of Operational Research. 208(2011) 19-27.

[3] A. Banerjee, Production lot sizing with work in process considerations in response to periodic demand, Int. J. Prod. Econ. 26 (2) (1992) 145-151.

[4] Chiang Kao, Wen-Kai Hsu, A Single period Inventory model with fuzzy demand, Computers and Mathematics with Applications. 43(2002) 841-848.

[5] H. J. Zimmermann, Fuzzy linear programming with several objective functions, Fuzzy sets and Systems. (1978) 146-55.

[6] K. Atanassov, G.Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy sets and Systems. 31(3) (1989) 343-349.

[7] K. Atanassov, Intuitionistic fuzzy sets and system. Fuzzy sets and Systems**.** 20(1986) 87-96.

[8] Liang- Yuh Ouyang , Hung- Chi Chang, A Minimax distribution free procedure for mixed inventory models involving variable lead time with fuzzy lost sales, Int.J. Prod. Eco. 76(2002) 1-12.

[9] L. A. Zadeh, Fuzzy sets, Information and Control. 8(1965) 338-356.

[10] M. Saleh, R. Oliva, C.E. Kampmann, Pal I. Davidson, A Comprehensive Analytical approach for policy Analysis of system dynamics models, European Journal of Operational Research. 203(2010) 673-683.

[11] P.P. Angelov, Optimization in an intuitionistic fuzzy environment, Fuzzy sets and Systems. 86(1997) 299-306.

[12] P. M. Ghare, G. P. Schrader, A model for exponentially decaying inventory, J. Industrial Engineering. 14(1963) 238-243.

[13] R.B. Misra, Optimum production lot size model for a system with deteriorating inventory, Int. J. Production Research. 13(1975) 495-505.

[14] R.E. Bellman, L.A. Zadeh, Decision making in a fuzzy environment, Management Science. 17(1970) B141-B164.

[15] R. P.Covert, G. C. Philip, An EOQ model for items with Weibull distribution deterioration, AIIE Trans. 53(1973) 23-332.

[16] S.Banerjee, T.K. Roy, Probabilistic inventory model with fuzzy cost components and fuzzy random variable, International Journal of Computational and Applied Mathematics. 5 (4) (2010 a) 501-514.

[17] S. Banerjee, T.K. Roy, Solution of a probabilistic fixed order interval system : a general Fuzzy Programming Technique and intuitionistic Fuzzy Optimization Technique, Global Journal of Finance and Management. 2 (2) (2010 b) 257-294.

[18] S.De , A. Goswami, A replenishment policy for items with finite production rate and fuzzy deterioration rate. Opsearch. 38(4) (2001) 419-430.

[19] T. L. Urban, An inventory model with an inventory level dependent demand rate and relaxed terminal conditions, Journal of Operational Research Society. 43(1992) 721-724.

[20] Yuh Ouyanga Liang, Kun-Shan Wub, Chia-Huei Ho, Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time, Int.J. Prod. Econ. 92(2004) 255-266.